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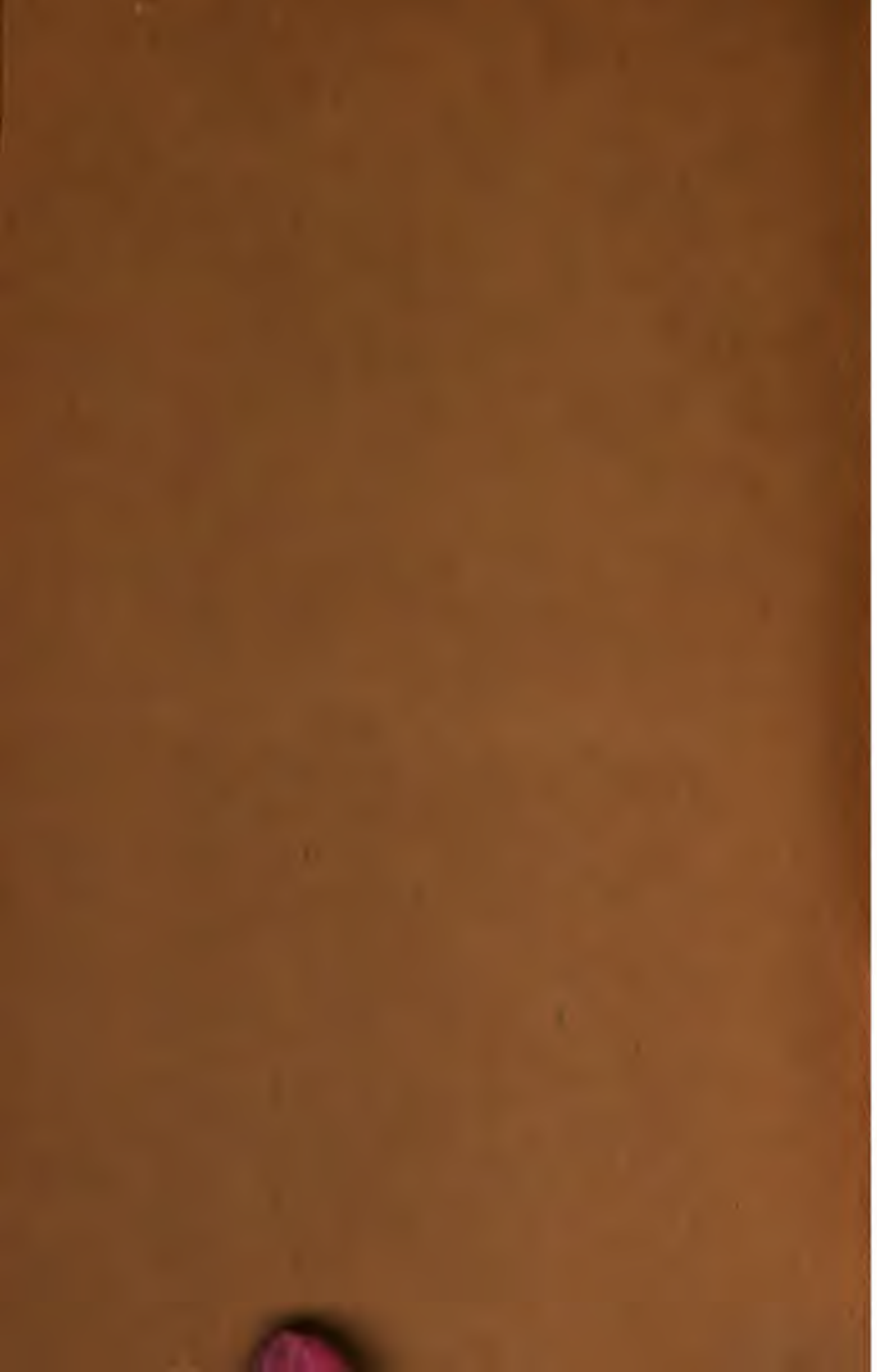

DEPARTMENT OF EDUCATION
LELAND STANFORD JUNIOR UNIVERSITY

PRESTON–STEVENS
STANDARD ARITHMETIC

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PRESTON-STEVENS

STANDARD ARITHMETIC

BY

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PREFACE

THE STANDARD ARITHMETIC is the second book of a series, yet is complete in itself and in many schools may be used throughout all the grades, if such use be preceded by a little preparatory work in number.

Attention is asked to the order of presentation, to the great number of practical problems, and to the applications of arithmetical processes to actual conditions arising in business and in all conditions of life. The teacher will find an unusual number and variety of problems and will note the sufficiency and value of the summaries. Some space is given to those simpler algebraic processes and geometric constructions which are now being tried out in the higher grades of elementary schools, and the introduction to certain phenomena of familiar science which are susceptible of mathematical solution or definition will appeal to the interest of many students. Sufficient space is given to Mensuration and to the Metric System.

Unnecessary or obsolete processes, tables, or facts have been eliminated.

The book is presented to progressive and earnest teachers in the belief that it will give to them

abundant material for the work of the several four or five higher years of the elementary school course and that it will appeal, in the nature or material of the exercises offered, and from which selection may be made, to the interests of their pupils.

NEW YORK, July, 1910.

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PRESTON–STEVENS

STANDARD ARITHMETIC

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CHAPTER I

NOTATION AND NUMERATION

INTEGRAL AND DECIMAL

1. The standards by which we count or measure are called **units.**

2. Numbers show *how many* units are expressed.

In 5, 9, 25, 36, etc., the unit is 1.

In 2 bushels, 8 bushels, 15 bushels, etc., the unit is 1 bushel.

In 4 feet, 9 feet, 36 feet, etc., the unit is 1 foot.

In 3 square feet, 6 square feet, etc., the unit is 1 square foot.

3. **Notation** is the method of writing numbers.

4. The Arabic notation uses the figures:

0	1	2	3	4	5	6
naught	one	two	three	four	five	six

7	8	9
seven	eight	nine

5. The Roman notation used the letters, I, V, X, L, C, D, M.

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

6. Numbers are written in words when they are very important, as the numbers thus written are not so easily changed.

In writing in the amount on a business paper, we write, one hundred fifty dollars, instead of $150. The figures 150 might be easily changed to 750.

7. **Numeration** is the method of reading numbers.

2425 is read, — two thousand, four hundred twenty-five.

8. In the use of the telephone and in calling numbers to others, we often name the figures in order, calling 2, 4, 2, 5 for 2425.

THE ARABIC SYSTEM

9. The Arabic system, which enables us to express all numbers, no matter how large, by the use of the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, is a system of grouping by tens, called the decimal system, and is the one in common use.

10. In writing whole numbers, the units are expressed by the right-hand figure, each figure space at the left having ten times the value of its next right-hand figure space. In 33 the right-hand figure 3 expresses 3 units; but the 3 in the next place expresses ten times 3, or 30 units. In 433, the 4 expresses $10 \times 10 \times 4$, or 400 units. 433 expresses 400 units, 30 units, and 3 units united into one expression, which we have learned to understand.

11. To read large numbers easily, we begin at units' place and group the figures into families or **periods,** and then give each period its family name as we read the number.

12. The names of these periods or families, in order from units' place toward the *left*, are ; **units, thousands, millions, billions,** etc.

In writing large numbers so that they may easily be read, we separate them into periods by use of commas.

We write 344487164 for reading, 344,487,164, and read it period by period; 344 million, 487 thousand, 164.

CAUTION. Do not separate numbers into periods while using them in computation. The comma which is used to separate periods is so nearly like the decimal point which is used to locate units' place, that it may be mistaken for it, if hastily made and used during computation.

13. Copy, point off into periods, and read aloud :

1. 34576, 453260, 432789, 40032, 435602, 435789.

2. 340678, 4023053, 8976420, 45320678, 45320679.

3. 670742034, 432506701, 806530435, 432870453261.

4. 3000045, 540000045, 53000060, 420000040, 3500032.

5. 400200300, 700800009, 500300002, 500000007.

14. In writing numbers in figures when you see them written in words or hear them read, write each period as if it stood alone, then place the comma and write the next period.

In writing fifteen thousand, four hundred seven in figures, we write 15 as if it stood alone, place the comma, then write 407. We have written 15,407, which is correct.

In writing thirty-five thousand, we write 35 as if it stood alone, place the comma, and as there are no units fill out the places with ciphers, thus, 35,000.

The ciphers, while they have no value of themselves, must be used to show the proper place for the other figures.

1, 10, 100, 1000, 10000, are very different numbers, as the ciphers used have pushed the 1 farther from units' place each time, and multiplied its value by ten each step it is moved to the left.

15. The **key** to the Arabic system is the *units' place.* The value of each figure in a number depends upon its distance from units' place.

16. Write in figures the following numbers:

1. Ninety-four thousand, six hundred fifty-five.

2. Eighty-four thousand, nine hundred seventy-five.

3. Two hundred eighteen thousand, five hundred sixty-seven.

Note to the Teacher. Changing from words to figures when the pupil reads the words is of little importance. Writing numbers in figures which one hears spoken is very important. Dictation exercises should be used freely.

NOTATION AND NUMERATION OF DECIMALS

17. Parts of numbers expressed in tenths, hundredths, thousandths, etc., are called **decimals.**

18. The principles of the Arabic system are used in writing and reading decimals.

In writing decimals, the decimal point (.) is used to tell where the units end in the number written. Units' place is the first place at the left of the decimal point. The location of units' place is the **key** to decimals as it is to whole numbers, and to the entire Arabic system.

In 365.24 the decimal point shows us that 5 is in units' place and stands as the *key* to the number. Each figure at its left has its value multiplied by ten for each step or place it is moved from the key position of units. Each figure at its right has its value divided by ten for each step or place it is removed from the key position of units.

The decimal part, therefore, like the integral part, is fixed by a law of place value of the figures, by a law of place values, etc. Each step of a figure to the left multiplies its value by ten, each step of a figure to the right divides its value by ten.

In the number given, 365.24, the 2 has one tenth of its unit value, as it is one place to the

right of units' place; the 4 has one hundredth of its unit value, as it is two places to the right of units' place. The 6 has ten times its unit value, as it is one place to the left of units' place; the 3 has one hundred times its unit value, as it is two places to the left of units' place.

This law is so important, as it is the **key** to all number expressions, that you should study it until you understand it thoroughly.

STUDY OF NUMBER EXPRESSIONS

19. Express fully what is meant by the following numbers:

1. 645.32.

(*a*) The 5 is in units' place and expresses 5 units.

(*b*) The 4 is one step to the left of units' place and expresses 10 times 4 units or 40 units.

(*c*) The 6 is two steps to the left of units' place and expresses 100 times 6 units or 600 units.

(*d*) The 3 is one step to the right of units' place and expresses $\frac{1}{10}$ of 3 units or $\frac{3}{10}$ units.

(*e*) The 2 is two steps to the right of units' place and expresses $\frac{1}{100}$ of 2 units or $\frac{2}{100}$ units.

2. 300.07.

(*a*) The 3 is two steps to the left of units' place and expresses 100 times 3 units or 300 units.

(*b*) The 7 is two places to the right of units' place and expresses $\frac{1}{100}$ of 7 units or $\frac{7}{100}$ units.

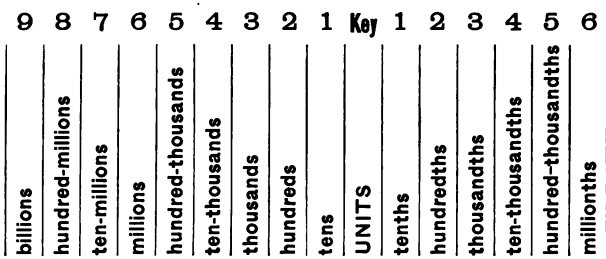
3. 45.18.

4. 8650.132.

5. 7006.008.

Note. While the decimal notation is not usually taken at this stage of the pupil's progress, it may well be studied here, if the units' place and not the decimal point is used as the key to number expressions. Careful study of the law of place values at this time will save much loss of time later, and will enable the pupil to associate the fundamental operations of integers and decimals. A little more careful study of the fundamental operations will thus enable the pupil to omit the entire chapter usually devoted to decimals.

20. The following chart shows the principles which govern the Arabic system of numbers, both integral and decimal.



21. To read the decimal part, we read the numbers in that part as if they were integers, then give them the group name of the place value of the last or right-hand figure. In reading aloud, we read the decimal point "and" to show its location.

1. Read 3415.016.

   3 thousand, 415 *and* 16 thousandths.

2. Read 42,678.143.

3. Read 30,250.0015.

4. Read 84,132,143.0875.

5. Read 360,204.00045.

READING AND WRITING NUMBERS WITH DECIMAL PARTS

22. Read the following numbers; then close your book and see if you can write them when you hear them read:

1.	3,462.125	1,432.024	136,743.0125
2.	43,250.0458	4,000.0005	400.00525
3.	23,456.0875	52,430.785	532,420.125
4.	34,523.0875	43,526.875	5,004.004
5.	23,452.0001	16,670.006	3,452.052

NOTATION AND NUMERATION OF U. S. MONEY

23. 1. The sign $ is used in place of the word "dollars," and is written before the number, thus $6, for six dollars.

2. Cents are hundredths of dollars; therefore, the last figure of cents must be in hundredths' place, the second place at the right of units.

3. The decimal point is used after dollars, which are the units numbered when the dollar sign is used.

Three dollars and five cents is written $3.05.

Ten dollars and forty-five cents is written, $10.45.

Eight cents is written \$.08, or, when written alone, 8 ¢.

24. Read the following; then close your book and see if you can write them when you hear them read:

1. \$80.15, \$65.04, \$750.10, \$753.25, \$7850.25, \$60.03.

2. \$150.04, \$250.43, \$601.15, \$78.02, \$6701.15, \$100.05.

3. \$243.50, \$6780.25, \$783.15, \$.54, \$.05, \$.09.

4. \$25,145.15, \$524,785.10, \$67,620.50, \$130,345.25, \$1.06.

5. \$60.02, \$600.15, \$707.07, \$606.06, \$5000.05, \$5200.52.

ROMAN NOTATION

25. The Roman notation is little used except to number clock dials, for dates on monuments, public buildings, etc., and to number the chapters in books. It is never used in computation.

1. The first nine numbers are:

I	II	III	IV	V	VI	VII	VIII	IX
1	2	3	4	5	6	7	8	9

2. The tens are:

X	XX	XXX	XL	L	LX	LXX	LXXX	XC
10	20	30	40	50	60	70	80	90

3. The hundreds are:

C	CC	CCC	CD	D	DC	DCC	DCCC	DCCCC
100	200	300	400	500	600	700	800	900

4. Thousands are written by placing M before the hundreds.

MC	MCC	MCCC	MCD	MD	MDC
1100	1200	1300	1400	1500	1600

MDCC	MDCCC	MDCCCC
1700	1800	1900

26. Read the following numbers:

XIX; XIV; XXV; XLV; XCIV; XXXV; XLVIII; CCCIV; LXXIX; MDCCCCIX; MDCCCCX; MDC; MDCCIV; LXVIII; XIX; XCIX; CDV.

Express the following numbers by the Roman notation:

15; 25; 46; 58; 105; 250; 64; 72; 48; 65; 45; 1150; 1200; 1909; 1906; 1910.

27. The Roman notation is based upon four principles as follows:

1. Repeating a letter, repeats its value.

2. If a letter is placed before one of greater value, its value is to be taken from the value of the greater.

3. If a letter is placed after one of greater value, its value is to be added to that of the greater.

4. If a letter is placed between two letters, each of greater value, its value is to be taken from their sum.

Note. Comparatively little time should be given to Roma notation, as it is rapidly passing from general use.

CHAPTER II

ADDITION

28. How many oranges are 3 oranges and 6 oranges?

How many apples are 2 apples, 4 apples, and 3 apples?

Paid 4¢ for a pencil, 5¢ for a pad, and 3¢ for some ink. How much did I pay in all?

There are 20 books on the shelf and 4 books on the table. How many books in both places?

If you put 4 splints with 6 splints, how many splints will you have?

29. **Addition** is the process of finding a number equivalent to two or more numbers.

30. The result obtained by addition is called the **sum**.

31. Only like numbers and units of the same order can be added.

32. The sign of addition is (+). It is called "plus," and means *more*.

Drill Exercises in Addition

33. Give the answers at sight:

3	5	4	6	8	9	4	2
+3	+4	+5	+3	+7	+8	+3	+8
—	—	—	—	—	—	—	—

4	5	6	2	6	7	7	8
+9	+6	+9	+8	+7	+8	+6	+8
9	6	8	7	9	7	6	8
+9	+7	+6	+8	+6	+9	+6	+9

ADDITION OF INTEGERS, UNITED STATES MONEY, AND DECIMALS

34. In writing numbers to be added, we must be careful to place units of the same order under each other. This is done by keeping the figures in units' place under each other, and by using uniform space between the figures of the numbers.

If we keep the figures in units' place under each other, we may write dollars and cents or numbers with decimals and add them just as we write and add whole numbers.

35. INTEGERS	U. S. MONEY	DECIMALS
3425	$34.25	342.5
604	6.04	60.4
3725	37.25	372.5
85	.85	8.5
7839	$78.39	783.9

1. In the first example shown, the column headed by 5 is in units' place. In the second, the one with U. S. money, the column headed by 4 is in units'

place. In the example where decimals are used, the column headed by 2 is in units' place.

2. When the decimal point is used in a number, units' place is always the first place at the left of the decimal point.

3. In addition of decimals, the right-hand figures of the numbers are not always in a vertical column; but the units' column must be vertical and the decimal points stand under each other.

$$\begin{array}{r} 3.462 \\ 17.14 \\ 8.3465 \\ \hline 28.9485 \end{array}$$

36. In adding, we begin at the right-hand figure, whether adding whole numbers or decimals. If there are more figures than one in the sum of any column, we write the right-hand figure in that column and carry the other figures to the next columns at the left. Business men, in adding long columns of figures, write the numbers carried at the top of the next columns, that they may test their work for any column, knowing what is to be included as "carried over" from the lower column.

We test addition by adding the columns both ways to see if we get the same sum.

The sum is always the same kind of units as the numbers added. Only like numbers can be added.

Exercises in Addition

37. Copy the following numbers and add them:

(1)	(2)	(3)
3468	$184.16	786.025
479	27.14	8.04
3467	17.92	19.365
14579	254.25	132.008
	$.	.

(4)	(5)	(6)
$187.86	375.014	$178.96
340.16	.018	18.48
27.15	3.7145	375.12
9.64	.0415	67.14
134.45	16.875	320.75

(7)	(8)	(9)
184325	87.645	345 bushels
78645	345.005	76
7869	1452.025	432
23456	345.0075	18
37829	42.0008	347
34526	145.005	17
457634	1453.2008	1345
		bushels

38. Write in columns and add:

1. 43768, 89765, 876432, 876, 8964, 4325, 4536, 43206, 4325.

2. $10.50, $165.28, $170.18, $650.24, $130.50, $750.20, $45.85.

3. 2345.78, 786.005, 879.652, .0785, 61.875, 64.025, .00875.

4. 6789, 5432, 79865, 67485, 96874, 897645, 87453, 87642, 3456.

5. $67.45, $750.18, $6.45, $850.75, $87, $120.15, $2.50, $.75.

6. 432.5, 678.008, 76.075, 68.0125, 600.008, 705.075, 76.0005.

7. $150.25, $45.76, $432.45, $78.15, $130.20, $200, $150.36, $105.05.

Special Drills to strengthen Addition

39. If you have trouble in adding rapidly and accurately, you will find the following drills helpful:

1. Learn these tables thoroughly:

4's	$+\frac{4}{4}$	$+\frac{5}{4}$	$+\frac{6}{4}$	$+\frac{7}{4}$	$+\frac{8}{4}$	$+\frac{9}{4}$
5's	$+\frac{5}{5}$	$+\frac{6}{5}$	$+\frac{7}{5}$	$+\frac{8}{5}$	$+\frac{9}{5}$	
6's	$+\frac{6}{6}$	$+\frac{7}{6}$	$+\frac{8}{6}$	$+\frac{9}{6}$		

7's	7 +7	8 +7	9 +7
8's	8 +8	9 +8	
9's	9 +9		

If you add these combinations both up and down, you have all of the combinations that will be likely to trouble you.

2. Then practice adding where you change the figure in tens' place, using the larger number first, as this is the form in which the combination will come to you in adding columns.

5	5	5	5	5	5	5	5	5	5	5	5
26	37	38	28	39	47	57	68	76	89	96	78
6	6	6	6	6	6	6	6	6	6	6	6
35	37	46	58	49	37	28	39	47	58	79	67
7	7	7	7	7	7	7	7	7	7	7	7
15	14	26	37	48	19	28	46	58	79	67	98
8	8	8	8	8	8	8	8	8	8	8	8
13	15	14	17	36	47	59	76	86	97	79	85
9	9	9	9								
16	18	14	13								

In adding 9, think of one less than 10 more, 43 and 9, 52; 87 and 9, 96, etc.

3. If you are very weak in addition and add very slowly, practice with a complete table in series with the combinations which trouble you. For example, if you find the addition of 8 troubles you, make a series table with 8, where you change the tens, —

8	8	8	8	8	8	8	8	8	8	8	8
15	25	35	45	55	65	75	85	95	105	115	125

If the combinations of 6's and 7's trouble you, make a table like this:

6	7	6	7	6	7	6	7	6	7
7	16	17	26	27	36	37	46	47	56

It is much better to find *your* trouble, and conquer it, than to continue general practice, hoping to overcome it in that way.

Practice adding as rapidly as possible. You will make more mistakes when you add slowly than when you add rapidly.

Business men place great importance upon the ability to add long columns of figures rapidly and accurately, so you may well study your personal need very carefully, and strive to overcome it by carefully selected drill exercises.

APPLICATIONS OF ADDITION

40. 1. How many acres in three fields containing 23 acres, 34 acres, and 38 acres?

2. A man bought a carriage for $135, a horse for $250, a whip for $1.75, and a robe for $12.75. What did they all cost?

3. A man owns three lots of the following sizes: 6.75 acres, 3.125 acres, 8.675 acres. How many acres are there in all?

4. Mr. Jones bought a farm for $3785. After spending $2900 for new buildings, he sold it at a gain of $750. For how much did he sell the farm?

5. The area of the United States is 3,602,990 square miles; of the Philippine Islands, 143,000 square miles; of Porto Rico, 3600 square miles; of Hawaii, 6740 square miles. What is the total area?

6. New York is 1405 miles east of Omaha by railroad, and San Francisco is 1864 miles west of Omaha by railroad. How far is it from New York to San Francisco by this route?

7. How many yards are there in three pieces of cloth, the first containing 18.375 yards, the second 41.625 yards, and the third 35.5 yards?

8. Find the amount to be paid for the following: tie, 50¢; gloves, $1.25; shirts, $2.50; umbrella, $3.50; rubbers, $.75; collars, 50¢.

9. In a city school there are 250 pupils in the first year, 275 in the second year, 234 in the third year, 194 in the fourth year, 187 in the fifth

year, 189 in the sixth year, 178 in the seventh year, and 157 in the eighth year. How many pupils in the school?

10. At an auction the following named sales are made: stove for $15; chairs for $12.75; rugs for $45; pictures for $15.75; piano for $275; books for $24.50; table for $3.75; writing desk for $10.50; sofa for $7.50. What was the total amount of the sales?

11. The railroad distance from New York to Albany is 144 miles; from Albany to Buffalo, 298; from Buffalo to Cleveland, 183; from Cleveland to Toledo, 109; from Toledo to Springfield, 365; from Springfield to St. Louis, 95 miles. What is the distance from New York to St. Louis by this route?

12. A man commenced farming in the West, and raised the first year 724 bushels of corn; the second year 3498 bushels; the third year 9872 bushels; the fourth year 9964 bushels; the fifth year 11078 bushels. How many bushels did he raise in the five years?

13. How many men in an army consisting of 52714 infantry, 5110 cavalry, 6250 dragoons, 3927 light-horse, 928 artillery, 250 sappers, and 406 miners?

14. A fruit dealer in Florida shipped for New York in one week 2464 boxes of oranges, 1632

boxes of pineapples, 1948 boxes of lemons, and 850 boxes of cocoanuts. What was the entire number of boxes shipped?

15. The area of Maine in square miles is 33040; of New Hampshire, 9305; of Vermont, 9565; of Massachusetts, 8315; of Rhode Island, 1250; of Connecticut, 4990. What is the area of the New England states in square miles?

CHAPTER III

SUBTRACTION

41. 1. What is the difference between 5¢ and 3¢?

The answer to the above question may be found in two different ways:

1st. By adding to the smaller number until it equals the larger, as business men do in making change.

2d. By taking away the smaller number from the larger and noting what remains.

2. Mary is 13 years old and her brother is 8 years old. What is the difference in their ages?

8		13
+5 difference	or	−8
13		5 difference

3. A boy had 24 marbles and lost 16 of them. How many had he left?

24 at first	24 at first	24
−16 lost	−16 lost	−16
? left	+8 left	8 *Ans.*

4. A man had 15 bushels of potatoes and sold 5 bushels. How many bushels had he left?

15	15	15
−5	5	−5
?	+10	10 *Ans.*

5. Bought coffee for 75¢ and gave the clerk $1. How much change should I receive?

$1.00
−.75
$?

The business man has learned to add to the 75¢ as he counts out the change, until he makes $1.

This method is much easier to learn, and also one by which we are much less likely to make mistakes. Begin at the 5, and reading down add as you read, saying 5 and 5, writing in the 5, are 10; then carry as in addition, and say, 8 and 2, writing the 2, make 10. You have written in 25, which is the right change.

6. Mr. Smith bought 18 tons of coal. How much was left after he had used 7 tons?

Question form:

18
− 7
?

Answer form:

18
− 7
11

Proof: 7 tons that he used + 11 tons left make 18 tons, which he had at first.

7. From 925 subtract 476.

925
− 476
449

By the old-fashioned plan of subtraction, this was a long and hard example. It was explained as follows: "The subtrahend is written under the minuend, and we begin at the right to subtract.

"Since 6 units cannot be subtracted from 5 units, one of the tens, which is equal to 10 units, is united with the 5 units, making 15 units. 6 units from 15 units leave 9 units, which are written under the units.

"Since *one* of the tens was united with the units, there is but 1 ten left. Because 7 tens cannot be subtracted from 1 ten, 1 hundred, which is equal to 10 tens, is united with the 1 ten, making 11 tens. 7 tens from 11 tens leave 4 tens, which are written under the tens.

"Since *one* of the hundreds was written with the tens, there are but 8 hundreds left. 4 hundreds from 8 hundreds leave 4 hundreds, which are written under the hundreds. Hence the remainder is 449. Proof: 449, the remainder, plus 476, the subtrahend, equals 925, the minuend. Hence the result is correct."

By the additive plan, we work as follows:

925
− 476

Begin at the right-hand figure of the lower number, and read down, adding in

enough to make the larger number at the top. We read, 6 and 9 are 15, writing in the 9 and carrying the 1 as in addition; 8 and 4 are 12, writing in the 4 and carrying the 1; 5 and 4 are 9. We prove our work by adding from the bottom, 9 and 6 are 15; 5 and 7 are 12; 5 and 4 are 9.

Note. The additive plan for subtraction is of special value, as it constantly strengthens addition. It has been carefully tested in the class room and in business and found to be the easiest and surest way of subtracting.

42. Copy, subtract, and prove:

(1)	(2)	(3)	(4)	(5)	(6)
764	876	765	4378	894	802
−321	−343	−342	−246	−370	−502
(7)	(8)	(9)	(10)	(11)	(12)
$678	$789	$786	$978	$769	$879
−435	−423	−421	−235	−425	−213
(13)	(14)	(15)	(16)	(17)	(18)
842	931	735	912	841	927
−376	−287	−368	−196	−267	−389

43. In writing decimal parts for subtraction, write the decimal points under each other, as this will place units of the same order in the same vertical columns. The operation of subtraction is the same as that of whole numbers.

Copy, subtract, and prove:

(1)	(2)	(3)	(4)
786.0158	67.005	89.0125	76.875
−67.0125	−8.024	−8.875	−7.325
.	.	.	.
(5)	(6)	(7)	(8)
78.325	74.012	76.75	27.375
−24.785	−53.005	−3.0125	−7.125
(9)	(10)	(11)	(12)
46.0126	16.0875	34.	75.
−6.0125	−5.005	−5.75	−8.125
(13)	(14)	(15)	(16)
$75.	$45.	$18.	$85.
−6.25	−8.75	−3.50	−2.75

44. 1. Only like numbers may be subtracted.

2. In subtraction, the larger number is called the *minuend*, the smaller number the *subtrahend*, and the result the difference or *remainder*.

3. The sign of subtraction is a short horizontal line (−). It is called "minus," and means *less*.

4. In the subtraction of decimals, if there are less decimal places in the minuend than in the subtrahend, we annex ciphers to the right of the decimal places of the minuend. This does not alter the value of the minuend.

5. Subtraction is the converse process of addition.

APPLICATION OF ADDITION AND SUBTRACTION

45. 1. A man bought some goods for $750 and sold them for $825. How much did he gain?

2. Bought some hay for $65, some straw for $10, and paid cash $50. How much do I still owe?

3. Bought a farm for $3500, paid $750 for improvements, and then sold it for $5000. How much did I gain?

4. Bought shoes for $2.50, a hat for $2.75, a suit of clothes for $25, a tie for $.75, and gave the man $35. How much change should I receive?

5. Mr. Smith had $1000 in the bank. He drew out $250 to pay his rent and $75 to buy some sheep. How much had he left in the bank?

6. From a farm containing 50 acres, 12.5 acres were sold. How many acres remained?

7. A man had 100 sheep, which he kept in three fields. There were 25 sheep in the first field, 35 in the second. How many sheep in the third field?

8. Some boys wish to raise $10 for a ball club. They have $7.50. How much more do they need to raise?

9. I owe Mr. Smith $500 and have $350. How much must I draw from the bank that I may pay him? If I had $400 in the bank, how much will be left after I have paid Mr. Smith?

10. A merchant gave his note for $5200. He paid at one time $2500 and at another $1525. How much remained to be paid?

11. Borrowed from my neighbor at one time $150, at another $75, and at another $125. Having paid him $275, how much do I still owe?

12. Mr. Smith owns 1215 sheep, Mr. Jones owns 150 more than Mr. Smith. How many sheep do both gentlemen own?

13. A merchant had 7520 barrels of flour. He sold 1200 barrels to one man, 750 to another, and 1350 to another. How many barrels of flour had he left?

14. A farmer raised 1575 bushels of wheat and 900 bushels of corn. After using 750 bushels of wheat and 350 bushels of corn, how many bushels of each remained?

15. A man sold some hay for $50, some grain for $25, and some potatoes for $12.50. With the money received he bought a wagon for $65. How much money had he left?

16. There are 15 acres in a field. After a man has plowed 10.25 acres, how much remains to be plowed?

17. A man measures the sides of his garden, and finds them to be 40.5 feet, 25.25 feet, 40.25 feet, and 24.75 feet. How many feet of fence will it take to fence this garden?

CHAPTER IV

MULTIPLICATION

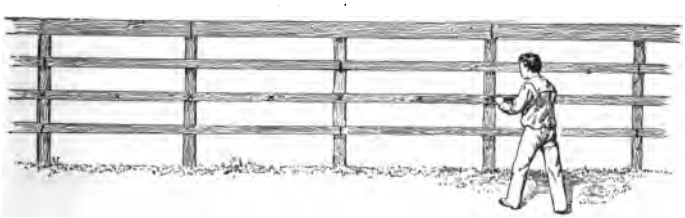
46. The process of taking one quantity as many times as there are units in another is called multiplication.

47. The sign of multiplication is an oblique cross ($\times$). It means that the numbers between which it is written are to be multiplied together.

INTRODUCTION TO MULTIPLICATION

48. 1. A boy measures a fence with a stick 4 feet long. He finds that the fence is 20 times as long as his stick. How long is the fence?

$$\begin{array}{r} 4 \text{ feet} \\ \times\, 20 \\ \hline 80 \text{ feet} \end{array} \qquad \textit{Ans.}\ 80 \text{ feet.}$$



2. A man takes a pailful of water 4 times from a tank of water. If the pail holds 10 quarts, how much water does he take from the tank?

10 quarts
× 4
40 quarts *Ans.* 40 quarts.

3. A man earns $2 each day for 6 days. How much does he earn?

$2
× 6
$12 *Ans.* $12.

4. In selling some cloth, a man measures off 1 yard 18 times. How much cloth does he measure?

5. A surveyor uses a tape 100 feet long to measure the length of a field. He uses the measure 15 times. How long is the field?

6. A man pays 3 $5 bills for an overcoat. What did the coat cost?

49. The numbers in multiplication have names, as follows:

$5 multiplicand
× 3 multiplier
$15 product

1. The multiplicand is the unit of measure.

2. The product is the same kind as the multiplicand.

3. The multiplier is always an abstract number.

APPLICATION OF MULTIPLICATION

50. 1. What will 9 pairs of shoes cost at $3.50 a pair?

2. A carpenter earns $ 24 a week. How much will he earn in 5 weeks?

3. There are 5280 feet in one mile. How many feet in 8 miles?

4. Find the cost of 24 pounds of butter at 32¢ a pound.

$ .32
× 24
128
64
$ 7.68

Multiply by each figure of the multiplier separately. Write the first figure obtained as a part of the product directly under the figure of the multiplier which you are using. Add the partial products for the complete product.

5. Find the cost of 165 acres of land at $ 175 an acre. (The unit of measure is $ 175, which is repeated 165 times.)

6. Find the cost of 75 pounds of butter at 36¢ a pound. (The unit of measure is 36¢. The multiplier is 75, an abstract number.)

7. Find the cost of 320 sheep at $ 3 each.

$ 3
× 320
$ 960

In the solution of problems, it is better **first** to determine the terms of the answer, and then make the computations in abstract form, to answer the question, "how many?"

For example:

320
× 3
960

Question. How many $——?
Answer. $ 960.

8. There are 3 feet in one yard. How many feet in 1760 yards?

$$\begin{array}{r} 1760 \\ \times\ 3 \\ \hline 5280 \end{array}$$

Question. —— feet?

Answer. 5280 feet.

9. Find the cost of 300 acres of land at \$ 75 an acre.

Long Way	Short Way
\$ 75	\$ 75
× 300	× 300
00	\$ 22500
00	
225	
\$ 22500	

By writing the ciphers at the right, we shorten the process, and still maintain the place values.

10. Find the cost of 800 pounds of butter at 32¢ a pound.

11. Find the cost of 300 pounds of cheese at \$.16 a pound.

12. Find the cost of 1000 bushels of potatoes at \$.55 a bushel.

13. Find the cost of 2000 bushels of wheat at \$ 1.12 a bushel.

MULTIPLICATION OF DECIMALS

51. 1. The process of multiplication of decimals is the same as that of the multiplication of whole numbers. The pointing off of the decimal places in the product is the only matter requiring special attention.

2. Study the following exercises and see if you can determine how the pointing off of the product is done.

1324	$13.24	1.324	1324	1324	13.24
× 5	× 5	× 5	× .5	× .05	× .05
6620	$66.20	6.620	662.0	66.20	.6620

3. To multiply where there are decimal places in one or both factors. Multiply as in whole numbers and point off as many decimal places in the product as there are decimal places in **both** factors.

4. Find the cost of 14.25 pounds of cheese at $.16 a pound.

14.25
.16
8550
1425
2.3800

Question. $——?

Answer. $2.38.

The terminal ciphers may be disregarded, as they have no value.

Drills in Multiplication

If you have any trouble with the tables of multiplication, study the following combinations carefully, reading them both ways. Read 2 3's are 6, and 3 2's are 6.

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline \end{array}$$

52.

$$\begin{array}{r} 3 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ \times\ 8 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ \times\ 9 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \times\ 8 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \times\ 9 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ \times\ 8 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ \times\ 9 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ \times\ 9 \\ \hline \end{array}$$

Written Exercises

53. 1. 7689×376.

2. 9678×679.

3. 3786×769.

4. 38976×697.

5. $\$476.75 \times 67$.

6. $\$896.75 \times 96$.

7. 314.25×73.

8. 61.78×6.9.

9. 76.89×9.67.

10. $350.65 \times .007$.

11. $\$76.45 \times 14.5$.

12. $\$350.75 \times 24.25$.

13. .75 bushels $\times$ 80.

14. 3.125 acres $\times$ 12.

15. Cost of 3.125 acres at \$80 an acre?

16. Total cost of 14 pounds at \$.75; 8 bushels at \$1.25; 15 pounds at \$.32; 1.45 bushels at \$1.50?

REVIEW OF ADDITION, SUBTRACTION, AND MULTIPLICATION

54. 1. Bought 4 lb. butter at 35¢; 12 lb. sugar at 6.5¢. Gave the clerk \$5. How much change should I receive?

2. A man sold 150 bu. of potatoes at 75¢ a bushel and bought a plow for \$15. How much money had he left?

3. A man having 150 sheep sold 40 of them at \$5 a head; 60 of them at \$5.50 a head, and the remainder at \$4.75 a head. How much did he receive for all of the sheep?

4. A dealer bought 50 bushels of apples at 85¢ a bushel. He sorted them and picked out 10 bushels of choice apples which he sold at $1.50 a bushel; 25 bushels of seconds which he sold at $1.10 a bushel, and sold the remainder at 95¢ a bushel. How much did he gain?

5. From the sum of 8614 and 316.25 take the sum of 360.75 and 350.5.

6. Multiply 378.55 by .04.

CHAPTER V

DIVISION

55. Division is the process of finding how many times one number or quantity contains another.

Division is the converse process of multiplication.

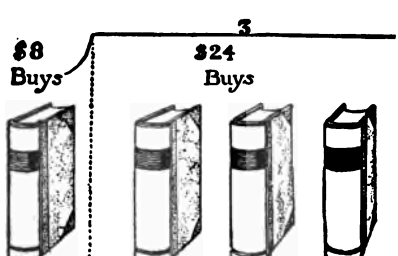
In answering the question, "7 3's are how many?" we use multiplication, and answer 21.

In answering the question "21 is how many 3's?" we use division and answer 7.

1. At \$8 each how many dictionaries can be bought for \$24?

Oral answer: As many dictionaries as the number of times \$24 contains \$8. *Ans.* 3.

Written form: $\$8\overline{)\$24}^{3}$ Proof: $\begin{array}{r}\$8\\ \times 3\\ \hline \$24\end{array}$



2. A school paid \$96 for desks at \$3 each. How many desks were bought?

Question. How many? 32

Answer. 32. $3\overline{)\$96}$

3. If 4 desks cost \$16, how much will each desk cost?

Question. How much? \$4

Answer. \$4. $4\overline{)\$16}$

56. The names used for the terms in division are shown in the following: Divide 12 by 4.

3 quotient

divisor $4\overline{)12}$ dividend

1. The divisor corresponds to one of the factors in multiplication and the quotient to the other.

2. The dividend corresponds to the product in multiplication.

3. Either the divisor or the quotient may be the factor which corresponds to the multiplier. The one that does correspond to the multiplier must be an abstract number.

4. If the whole of the dividend is not divided, the part that is left is called the remainder.

5. If there be a remainder, and we wish to show the complete quotient, we write the remainder over the divisor in fractional form at the end of the quotient. This shows that that part of the dividend has not yet been divided.

Divide 5 by 2. *Ans.* $2\frac{1}{2}$.

In the study of fractions, a few pages further on, we shall see just what these fractional forms mean, and how they may be changed into other forms.

6. Another common way of adjusting this remainder is to annex ciphers, and carry on the division, giving the quotient a decimal form.

Divide 5 by 2. *Ans.* 2.5.

Note. In division, many people begin with the divisor, and think "2 is contained in 6 how many times?" instead of "6 contains how many 2's?" You may use either form for the following exercises, but we fully recommend the use of the form which begins with the dividend. If you were using objects, you would begin with the pile of 6, and take out 2 as many times as you could until no more 2's remained. Not only is the form which begins with the dividend more logical, but it makes the work of actual computation much easier, as will be seen later.

APPLICATIONS AND EXPLANATIONS

57. 1. At \$3 each how many desks can be bought for \$804?

268 *Question.* How many? Think, 8 is 2 3's
\$3)\$804 *Answer.* 268. and 2; 20 is 6
3's and 2; 24 is
8 3's. (The answer is 268, not 268 desks. It corresponds to the multiplier, is an abstract number, and answers the question "how many" and not the question "what?" In the proof we must have the 268 in abstract form. Proof: \$3 × 268 = \$804.)

2. A man paid $784 for 8 acres of land. How much did he pay an acre?

$$\begin{array}{r} \$98 \\ 8)\overline{\$784} \end{array}$$

Question. How much? *Answer.* $98.

Proof:
$$\begin{array}{r} \$98 \\ \times 8 \\ \hline \$784 \end{array}$$

3. Divide 7834 by 9.

$$\begin{array}{r} 870\frac{4}{9} \\ 9)\overline{7834} \end{array}$$

Think, 78 is 8 9's and 6; 63 is 7 9's; 4 is no 9 and 4 remainder.

4. Divide 2944 by 4.

SHORT DIVISION

$$\begin{array}{r} 736 \\ 4)\overline{2944} \end{array}$$

LONG DIVISION

$$\begin{array}{r} 736 \\ 4)\overline{2944} \\ \underline{28} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \end{array}$$

When the numbers are large and we cannot do the work mentally, we write it out in full in a form that is commonly called "long divison."

5. Divide 11904 by 31.

$$\begin{array}{r} 384 \\ 31)\overline{11904} \\ \underline{93} \\ 260 \\ \underline{248} \\ 124 \\ \underline{124} \end{array}$$

Proof:
$$\begin{array}{r} 384 \\ \times 31 \\ \hline 384 \\ \underline{1152} \\ 11904 \end{array}$$

58. The four steps of long division are:

1. See how many times the divisor is contained by the first part of the dividend.

2. Write the number of times directly over the last figure of the part of the dividend used.

3. Multiply the divisor by this number, and subtract the product from the part of the dividend used.

4. Annex the next figure of the dividend and proceed with the division.

Note. By writing the quotient figure directly over the last figure of the part of the dividend used we maintain the place value of the figures used. In long division the chief difficulty rests with the determination of the number of times the dividend contains the divisor.

Exercises in Division of Integers, "Short" and "Long" Forms

59. 1. Divide by 3: 4524; 60459; 7401; 76452; 876321; 603245.

2. Divide by 4: 67824; 45286; 43210; 42396; 457863; 456780; 3002.

3. Divide by 5: 6730; 57865; 43215; 67890; 45762; 432106; 896043.

4. Divide by 6: 6780; 7860; 453218; 423569; 4576321; 43895624.

5. Divide by 7: 6742; 45321; 43256; 4578932; 436275; 8764524.

6. Divide by 8: 678042; 45760; 45326; 478954; 4578632; 45768934.

7. Divide by 9: 678213; 453267; 43789; 452301; 426015; 768324.

8. Divide by 21: 6784; 45320; 43268; 45326; 43789; 43268.

9. Divide by 51: 6780; 768420; 453789; 43216; 423567; 476324.

10. Divide by 72: 67840; 432678; 457869; 43216; 425368; 423165.

11. Divide by 812: 4325678; 4326750; 456897324; 456782; 4002438.

12. Divide by 29: 36748; 45326; 43286; 45326; 43876; 4235601.

13. Divide by 729: 347625; 452367; 43876; 487632; 4532106.

DIVISION OF UNITED STATES MONEY

60. 1. Divide \$432.15 by 5. *Ans.* \$86.43.

$$\begin{array}{r} \$\,86.43 \\ 5)\overline{\$\,432.15} \end{array}$$

Proof: \$86.43 × 5 = \$432.15

2. Divide \$432.15 by \$.05 *Ans.* 8643.

$$\begin{array}{r} 8643 \\ \$.05)\overline{\$432.15} \end{array}$$

Proof: \$.05 × 8643 = \$432.15

3. Divide \$432.15 by \$5. *Ans.* 86.43.

$$\begin{array}{r} 86.43 \\ \$\,5)\overline{432.15} \end{array}$$

Proof: \$5 × 86.43 = \$432.15

61. The process of division in U. S. money and in decimals is the same as in whole numbers. The location of the decimal point in the quotient may be determined in either of two ways, as follows:

1. Remembering that division is the converse operation of multiplication, we may make the number of decimal places in the divisor and the quotient equal those in the dividend.

2. If the divisor is an integer, we may locate the decimal point directly over the decimal point of the dividend.

When the divisor is not given in integral form, we may change **both** dividend and divisor so that the divisor will be an integer, by moving the decimal point the same number of places in **both** divisor and dividend. This will not change their relative values, nor the value of the quotient. Select the rule which you understand, and use it.

DIVISION OF DECIMALS

Exercises and Explanations

62. 1. Divide 34.75 by 25. *Ans.* 1.39.

```
      1.39
25)34.75
   25
   97
   75
  225
  225
```

Location of decimal point in the quotient:

1st plan. Two places in the dividend, none in the divisor, there must be two in the quotient.

2d plan. Divisor is

an integer, the point in the quotient must be directly over that in the dividend.

2. Divide 7.8125 by 31.25. *Ans.* .25.

```
      .25
31.25)7.8125
      6 250
      1 5625
      1 5625
```

1st plan. Four places in the dividend, two in the divisor, therefore, two in the quotient.

2d plan. Moving the point two places to the right in both dividend and divisor makes the divisor a whole number and locates the point in the dividend between the 1 and the 2 of the dividend. The point in the quotient is placed directly over this.

3. Divide 625 by .025. *Ans.* 25000.

```
   25 000
.025)625.000
     50
     125
     125
```

By the second plan of locating the decimal point in the quotient, the movement of the point three places to the right to make the divisor an integer demands the writing of the three ciphers.

By the first plan, we remember that the number of decimal places in the dividend, corresponding to the product in multiplication, must be equal to the number of decimal places in **both** factors; and as there are three places in one of the factors,

the dividend must have at least three places, which we form by annexing the three ciphers.

Exercises in division of decimals
(Prove your answers.)

63. 1. Divide .00335 by 6.7.

2. Divide 335 by .067.

3. Divide 3.35 by 67.

4. Divide .0255 by .051.

5. Divide 255 by .051.

6. Divide 2.55 by 0.51.

7. Divide 25.5 by 0.51.

8. Divide 25.5 by .051.

9. Divide .00255 by 51.

10. Divide .014532 by .0692.

11. Divide .04905 by .327.

12. Divide 6.2512 by .37.

13. Divide 75 by .0125.

14. Divide 105.70 by 3.5.

Short process, where there are terminal ciphers in the divisor

64. 1. Divide 50 by 10. (Change 50. to 5.0.)
Ans. 5.

2. Divide 500 by 100. (Change 500. to 5.00.)
Ans. 5.

3. Divide 5000 by 1000. (Change 5000. to 5.000.) *Ans.* 5.

4. Divide 843.75 by 10. (Move decimal point one place to the left, changing 843.75 to 84.375.) *Ans.* 84.375.

5. Divide 843.75 by 100. (Move point two places to left.) *Ans.* 8.4375.

6. Divide 843.75 by 1000. (Move point three places to the left.) *Ans.* .84375.

7. Divide 578.345 by 100.

8. Divide 67.895 by 1000.

9. Divide 678.5 by 1000.

10. Divide 6 by 1000.

11. Divide 1200 by 200. (Move point two places to the left, and divide by 2.)

12. Divide 18000 by 3000. (Move point three places to the left and divide by 3.)

13. Divide 8 by 2000.

14. Divide 1480 by 2000.

15. Divide 14.875 by 500.

16. Divide 6780 by 2000.

17. Divide 76.024 by 8000.

18. Divide 84.008 by 200.

19. Divide 164.04 by 2000.

Finding the cost, when the price is given for hundreds, thousands, or tons, and the quantity expressed in units

65. 1. Find the cost of 465 pounds of sugar at $5.80 per C.

465 pounds = 4.65 C pounds.

$5.80 × 4.65 = $ ——. *Ans.*

2. Find the cost of 8400 lb. of coal at $6 per ton.

2000 lb. = 1 T.

8400 lb. = 4.2 T.

$6 × 4.2 = $ ——. *Ans.*

3. Find the cost of 8500 bricks at $9 per M.

8500 = 8.5 M.

$9 × 8.5 = $ ——. *Ans.*

4. What will 875 pineapples cost at $13 per C?

875 = 8.75 C.

$13 × 8.75 = $ ——. *Ans.*

5. Find the cost of 7500 lb. of hay at $18 per ton.

6. Find the cost of 675 lb. of coal at $6 per ton.

7. Find the cost of 7500 shingles at $5 per M.

GENERAL REVIEW PROBLEMS

66. 1. If 365 bushels of corn grow on each acre, how many bushels will grow on 25 acres?

2. A man bought 28 tons of hay at $ 15 a ton and sold it at $ 18.50 a ton. How much did he gain?

3. How many bushels of onions at $.82 a bushel can be bought for $ 112.34?

4. If 46 tons of iron cost $ 3461.50, what will 8 tons of iron cost at the same rate?

5. Find the cost of 23475 board feet of pine lumber at $ 30 per M.

6. James runs 180 yards per minute and Frank pursues him at the rate of 200 yards per minute. How long will it take Frank to catch James if James has 2 minutes' start?

7. What number multiplied by 72084 will produce 5190048?

8. A man is 45 years old and has been married 19 years. How old was he when married?

9. The distance from Paris to Berlin is 1308 kilometers. A kilometer is .6214 of a mile. How many miles is it from Paris to Berlin?

10. If the average rate of a railway train is 36.7 miles per hour, how long will it take the train to run 1101 miles?

11. Find the cost of 200 bushels of potatoes at $.625 a bushel.

12. A man bought a piano for $ 276 and agreed to pay for it in monthly payments of $ 17.25 each.

How many months will it take him to pay for the piano?

13. A contractor purchased a million bricks. What did they cost him at $9 per M?

14. A grocer bought 250 pounds of coffee for $82.50 and sold it at $.38 a pound. How much did he gain?

15. If an acre of land cost $38.75, how many acres can be bought for $3560?

16. If .04 of the quantity of milk is cream, how many quarts of cream are there in 250 quarts of milk?

17. How much coal at $6 per ton can be bought for $75?

18. Find the cost of 12.5 tons of hay at $18.50 a ton.

19. How much is gained by selling molasses at 45 cents a gallon if 63 gallons cost $25?

20. I bought six loads of coal, each weighing 2345 pounds, at $5.50 a ton. What did the coal cost?

21. The difference between two numbers is 79486 and the larger number is 98979. Find the smaller number.

22. What is the total cost of 25 sheep if 17 of them cost $5.75 a head and the others $6.25 a head?

23. The product is 259740 and the multiplicand is 468. What is the multiplier?

24. Find the difference between .00783 and .0231.

SUMMARY

Summary of the Analysis of Number and the Fundamental Operations with Integers and Decimals

67. 1. Units are the standards by which we count or measure.

2. Numbers show how many units are expressed.

3. Notation is the process of writing numbers.

4. The Arabic notation uses figures to express numbers.

5. The Roman notation uses letters to express numbers.

6. Numeration is the method of reading numbers.

7. The Arabic system of notation is based upon units' place. The value of each figure in a number is dependent upon its location in the number. The value of figures in a number increases toward the left, and decreases toward the right of units' place by the scale of ten for each step.

8. In reading large numbers we group them into periods of three figures each, beginning at units' place. Commas are used to mark the separation of a number into periods for convenience in reading it.

9. Counting toward the left, these periods are named, units, thousands, millions, billions, etc.

10. At the right of units' place, in the decimal part of the number, we do not group into periods of figures, but give each figure its place name. These names are, tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, etc.

11. The decimal point is used after units' place to show where the whole number ends and the decimal part begins.

12. In reading numbers aloud which contain decimal parts, we read each group or period of whole numbers as if it stood alone, then give it the name of the place value of its last figure, before reading the next period. We call the decimal point "and," and then read the decimal part, giving it the name of the place value of its last figure.

13. In U. S. money the sign $ is used for dollars, and the decimal point is used between dollars and their decimal parts which are called cents.

14. Addition is the process of finding a number equivalent to two or more numbers. The result is called the sum.

15. The sign of addition is +. It is called "plus," and means *more*.

16. Only like numbers and units of the same order can be added.

17. The addition of U. S. money and of decimals is like the addition of whole numbers, care being taken to write units of the same order under each other.

18. Subtraction is the process of finding the difference between two like numbers.

19. The operation of subtraction may be performed, either by adding to the smaller number until it equals the larger, or by taking the smaller number from the larger.

20. In subtraction the larger number is called the minuend, the smaller number is called the subtrahend, and the result is called the difference or remainder.

21. The sign of subtraction is a short horizontal line (−). It is called " minus," and means *less*.

22. Only like numbers and units of the same order can be subtracted.

23. The process of subtraction of U. S. money and of decimals is like that of whole numbers, care being used to write units of the same order under each other.

24. Multiplication is the process of taking one quantity as many times as there are units in a number called the multiplier. The quantity repeated or used as the unit of measure is called the multiplicand.

25. The sign of multiplication is the oblique cross ($\times$). It means that the numbers between which it is written are to be multiplied together.

26. The result obtained by multiplication is called the product. It always names the same kind of units as the multiplicand.

27. In multiplication of U. S. money and of decimals, the process is like that of whole numbers. We fix units' place in the product by pointing off as many decimal places in it as there are decimal places in both factors.

28. Division is the converse operation of multiplication. It is the process of finding how many times one number or quantity, called the dividend, contains another number or quantity, called the divisor. The sign of division is $\div$. When used at all, it is used after the dividend and before the divisor, thus $12 \div 3 = 4$. $\$18 \div \$3 = 6$.

29. The dividend in division corresponds to the product in multiplication. The divisor and the quotient correspond to the factors in multiplication. The one used as the unit of measure corresponds to the multiplicand. The one which is used in abstract form corresponds to the multiplier. Both the quotient and the divisor may be abstract, and one of them must be used in abstract form.

30. In the division of U. S. money and decimals, we proceed as in division of whole numbers, point-

ing off the quotient so that the number of decimal places in the divisor and in the quotient shall equal the number of decimal places in the dividend.

31. To shorten the process of multiplying or dividing by 10, 100, 1000, etc., we may get the required result at once by changing units' place in the number multiplied or divided. This is done by moving the decimal point toward the right for multiplication and toward the left for division as many places as there are tens, or ciphers, in the multiplier or divisor.

32. We prove the result of our work in the fundamental operations as follows: In addition, we add the columns the other way. In subtraction, we add the subtrahend and the result to see if their sum is equal to the minuend. In multiplication, we usually go over our work a second time. In division, we multiply the quotient and the divisor together, adding the remainder, if there be any, to see if the result is equal to the dividend.

33. In the solution of problems, we first determine the terms of the answer, then decide upon the necessary steps to answer the question "how many?" and perform the operations, using the numbers in abstract form.

CHAPTER VI

PROPERTIES OF NUMBERS

68. The following statements and definitions are written here as a convenient arrangement for reference. They need not be learned at one time.

1. An integral number or integer is a number expressing whole things.

2. The factors of a number are the integers which multiplied together will produce it. 3 and 4 are factors of 12.

3. An exact divisor of a number is an integer that will divide that number without a remainder.

4. A prime number is one that has no exact divisors except itself and one.

5. Numbers are prime to each other when they have no common integral divisor. 16 and 25 are prime to each other.

6. An even number is one that is exactly divisible by 2. 2, 4, 6, 8, 10, 12, 14, 16, etc., are even numbers.

7. An odd number is one that is not exactly divisible by 2. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, etc., are odd numbers.

8. A prime factor is a prime number used as a factor.

9. A common divisor of two or more numbers is a common factor of each of the numbers.

10. The greatest common divisor of two or more numbers is their greatest common factor.

11. A common multiple of two or more numbers is a number exactly divisible by each of them.

12. The least common multiple of two or more numbers is the least number exactly divisible by each of them.

13. Factoring is the process of separating a number into its factors.

The most important use of factoring in arithmetic is in cancellation, in reducing fractions to their lowest terms, and in addition and subtraction of fractions where we have to find the least common multiple of their denominators in order to reduce them to similar fractions.

14. Cancellation is the process of shortening the operations of division by canceling equal factors from numbers which are to be multiplied together to form the dividend, and from numbers which are to be multiplied together to form the divisor.

15. When several numbers are to be treated as one number, they are included within parentheses ( ).

$(5+3) \times 2$ means that the sum of 5 and 3 is to be multiplied by 2.

$(17-5)\div 3$ means that the difference between 17 and 5 is to be divided by 3.

When numbers are written in line, as $3\times 2+4\div 2$, the operations of multiplication and division are to be performed before the operations of addition and subtraction. The signs + and − separate such expressions into sections or terms.

CHAPTER VII

FRACTIONS

69. A fraction is one or more of the equal parts of a whole thing.

70. If a whole thing be divided into
2 equal parts, each part is called one half, written $\frac{1}{2}$.
3 equal parts, each part is called one third, written $\frac{1}{3}$.
4 equal parts, each part is called one fourth, written $\frac{1}{4}$.
5 equal parts, each part is called one fifth, written $\frac{1}{5}$.
6 equal parts, each part is called one sixth, written $\frac{1}{6}$.

71. If a whole thing is divided into 8 equal parts, and 3 of those parts are expressed, it is written $\frac{3}{8}$.

$\frac{4}{5}$ shows 4 of the 5 equal parts of some whole thing.

$\frac{2}{3}$ pound shows 2 of the 3 equal parts of one pound.

$\frac{6}{7}$ yard shows 6 of the 7 equal parts of one yard.

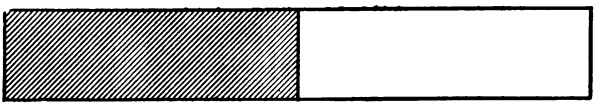
72. In a fraction the number which shows into how many parts the whole thing has been divided is called the denominator, as it names the parts.

73. In a fraction the number which shows how many of the parts are taken is called the numerator, as it numbers the parts.

74. The following diagrams picture the value of fractional expressions:

Shaded. 1 divided into 2 equal parts. Unshaded.

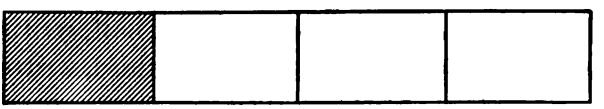
$\frac{1}{2}$ $\frac{1}{2}$



HALVES

Shaded. 1 divided into 4 equal parts. Unshaded.

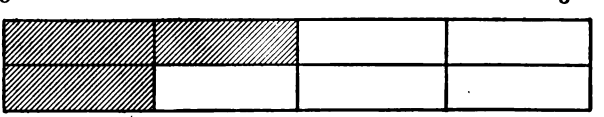
$\frac{1}{4}$ $\frac{3}{4}$



FOURTHS

Shaded. 1 divided into 8 equal parts. Unshaded.

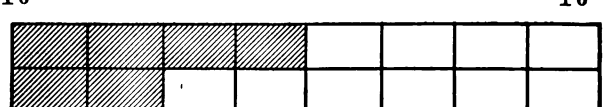
$\frac{3}{8}$ $\frac{5}{8}$



EIGHTHS

Shaded. 1 divided into 16 equal parts. Unshaded.

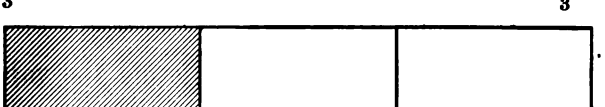
$\frac{6}{16}$ $\frac{10}{16}$



SIXTEENTHS

Shaded. 1 divided into 3 equal parts. Unshaded.

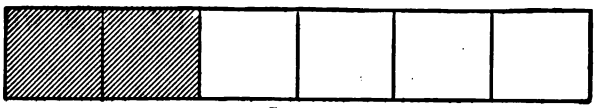
$\frac{1}{3}$ $\frac{2}{3}$



THIRDS

Shaded. 1 divided into 6 equal parts. Unshaded.

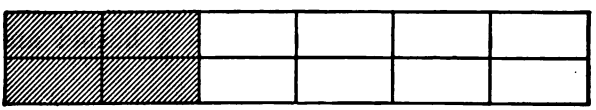
$\frac{2}{6}$ $\frac{4}{6}$



SIXTHS

Shaded. 1 divided into 12 equal parts. Unshaded.

$\frac{4}{12}$ $\frac{8}{12}$



TWELFTHS

STUDY OF FRACTIONAL EXPRESSIONS AND APPLICATIONS

Note. Before taking the following exercises, the pupils of the class should be supplied with small rectangular pieces of paper, which they may fold and crease and mark to help them in securing clear conceptions of what is given and what is required in each question. Large diagrams on the board should also be used to guide the work of the pupils. Use standard size for the type of one, that correct concepts may be secured in the comparison of the fractional forms. Careful study of these introductory exercises will enable the pupils to get a clear idea of fractional expressions, and make rapid progress possible when larger fractions are used in later exercises.

Oral Exercises in the Study of Fractional Expressions

75. 1. If $\frac{1}{2}$ pound of candy cost 12 ¢, what will 1 pound cost?

2. If $\frac{1}{2}$ bushel of potatoes cost 30 ¢, what will 1 bushel cost?

3. If $\frac{1}{4}$ pound of candy cost 10 ¢, what will 1 pound cost?

4. If $\frac{1}{4}$ pound of candy cost 6 ¢, what will $\frac{1}{2}$ pound cost?

5. If $\frac{1}{2}$ pound of candy cost 18 ¢, what will $\frac{1}{4}$ pound cost?

6. If $\frac{1}{2}$ pound of candy cost 20 ¢, what will $\frac{3}{4}$ pound cost?

7. What part of a bushel is $\frac{1}{2}$ bushel and $\frac{1}{4}$ bushel?

8. What part of a pound is $\frac{1}{4}$ pound and $\frac{1}{2}$ pound?

9. If $\frac{1}{2}$ pound cost 16 ¢, what will $\frac{2}{4}$ pound cost?

10. If $\frac{1}{2}$ bushel cost 30 ¢, what will $\frac{4}{8}$ bushel cost?

11. What will $\frac{1}{3}$ yard cost at 24 ¢ a yard?

12. What will $\frac{2}{3}$ pound cost at 18 ¢ a pound?

13. What will $\frac{1}{6}$ pound cost at 48 ¢ a pound?

14. What will $\frac{5}{6}$ bushel cost at 36 ¢ a bushel?

15. If $\frac{1}{6}$ pound cost 5 ¢, what will $\frac{1}{3}$ pound cost?

16. If $\frac{1}{6}$ pound cost 6 ¢, what will $\frac{1}{2}$ pound cost? $\frac{3}{4}$ pound?

17. If $\frac{1}{3}$ pound cost 9 ¢, what will $\frac{4}{12}$ pound cost?

18. If $\frac{1}{2}$ yard cost 15 ¢, what will $\frac{6}{12}$ yard cost? $\frac{3}{6}$ yard?

19. Sold $\frac{1}{2}$ of my farm. What part of the farm remained?

20. Lost $\frac{1}{3}$ of my marbles. What part of them remained?

21. Lost $\frac{1}{3}$ of my marbles and then had 12 marbles left. How many marbles had I at first?

22. After spending $\frac{1}{4}$ of my money I had \$ 30 left. How much had I at first?

23. A man, owning $\frac{1}{2}$ interest in a factory, sold $\frac{1}{2}$ of his share for \$ 5000. What was the value of the whole factory at that rate?

24. Two boys buy $\frac{1}{2}$ pound of candy at 40 ¢ a pound. If each paid an equal amount, what should each one pay?

25. What will $1\frac{1}{2}$ pounds of candy cost at 50 ¢ a pound?

26. What will $1\frac{1}{2}$ bushels of potatoes cost at 60 ¢ a bushel?

27. What will $3\frac{1}{4}$ quarts of peanuts cost at 8 ¢ a quart?

28. What will $5\frac{1}{3}$ pounds of coffee cost at 30 ¢ a pound?

29. What will $2\frac{1}{4}$ yards of ribbon cost at 20 ¢ a yard?

30. What will $4\frac{1}{8}$ quarts of berries cost at 8 ¢ a quart?

Expressing Fractional Values by Diagrams

76. 1. Show by diagram, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$.

2. Show by diagram, $\frac{1}{2}$ and $\frac{1}{4} = \frac{3}{4}$.

3. Show by diagram that $\frac{1}{5}$ is smaller than $\frac{1}{4}$.

4. Show by diagram that $\frac{2}{3}$ is larger than $\frac{2}{5}$.

5. Show by diagram that $\frac{5}{6}$ is larger than $\frac{2}{3}$.

6. Show by diagram that $\frac{2}{3} = \frac{6}{9}$.

7. Show by diagram that $\frac{4}{8} = \frac{1}{2}$.

8. Show by diagram that $\frac{3}{5} = \frac{6}{10}$.

9. Show by diagram that $\frac{5}{4} = 1\frac{1}{4}$.

10. Show by diagram that 2 and $\frac{1}{2} = \frac{5}{2}$.

Note. This exercise is only suggestive. Many more exercises may be outlined by the teacher. Free use of diagrams in the solution of problems is very helpful to the pupils.

REDUCTION OF FRACTIONS

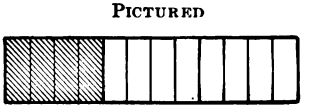
77. Reduction is the process of changing the form without changing the value.

Changing Fractions to their Lowest Terms, or Simplest Forms

78. 1. Reduce $\frac{4}{12}$ to its simplest form.

$$\frac{4 \div 4}{12 \div 4} = \frac{1}{3}.$$

PICTURED



Ans. $\frac{1}{3}$.

2. Reduce $\frac{8}{16}$ to its simplest form.

$$\frac{8 \div 8}{16 \div 8} = \frac{1}{2}.$$

Ans. $\frac{1}{2}$.

3. Reduce $\frac{16}{128}$ to its simplest form.

$$\frac{\overset{2}{\smile}}{16}{128} = \frac{\overset{2}{\smile}}{8}{64} = \frac{\overset{4}{\smile}}{4}{32} = \frac{1}{8}. \qquad Ans.\ \frac{1}{8}.$$

When we cannot see at once what large divisor to use, we may use several small divisors, as shown above.

The divisors 2, 2, and 4, used in this exercise, if multiplied together would give us the greatest common divisor, $2 \times 2 \times 4 = 16$.

If we divided by 16 at first, we should get the lowest terms at once, as follows:

$$\frac{16 \div 16}{128 \div 16} = \frac{1}{8}.$$

4. Reduce the following fractions to their lowest terms:

$\frac{24}{144}$; $\frac{18}{18}$; $\frac{63}{108}$; $\frac{128}{256}$; $\frac{72}{90}$; $\frac{36}{54}$; $\frac{84}{156}$.

79. An improper fraction is one whose numerator is larger than its denominator. It is, therefore, greater than one whole thing, and should be expressed as a mixed number.

80. Change the following improper fractions to mixed numbers:

1. $\frac{5}{2}$. $\frac{5}{2} = 2\frac{1}{2}$. *Ans.* (Show this by a diagram.)
2. Change $\frac{5}{4}$ to a mixed number. $\frac{5}{4} = 1\frac{1}{4}$. *Ans.*
3. Change $\frac{8}{3}$ to a mixed number. $\frac{8}{3} = 2\frac{2}{3}$. *Ans.*

Dividing the numerator by the denominator changes an improper fraction to a mixed number.

4. Reduce $\frac{15}{7}$ to a mixed number.
5. Reduce $\frac{27}{14}$ to a mixed number.
6. Change $\frac{125}{18}$ to a mixed number.
7. Change $\frac{160}{23}$ to a mixed number.
8. Change $\frac{72}{13}$ to a mixed number.
9. Change $\frac{150}{17}$ to a mixed number.
10. Change $\frac{250}{9}$ to a mixed number.

Changing Mixed Numbers to Improper Fractions

81. 1. Change $2\frac{1}{2}$ to an improper fraction.

$1 = \frac{2}{2}$
$2 = \frac{4}{2}$
$2\frac{1}{2} = \frac{5}{2}$

or $2\frac{1}{2} = \frac{2 \times 2 + 1}{2} = \frac{5}{2}$.

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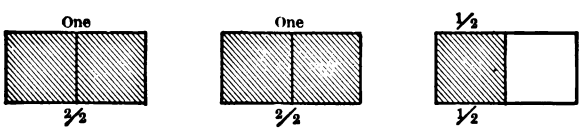

2. Change $8\frac{1}{3}$ to an improper fraction.
$8\frac{1}{3} = 8 \times 3$ plus 1 thirds $= \frac{25}{3}$.
3. Change $12\frac{1}{2}$ to an improper fraction.
4. Change $4\frac{7}{8}$ to an improper fraction.
5. Change $14\frac{7}{8}$ to an improper fraction.
6. Change $14\frac{2}{3}$ to an improper fraction.
7. Change $24\frac{4}{5}$ to an improper fraction.
8. Change $16\frac{3}{8}$ to an improper fraction.

ADDITION AND SUBTRACTION OF FRACTIONS

82. Fractions expressing parts of the same size are called similar fractions.

83. Fractions must be made similar before they can be added or subtracted.

Exercises in Addition and Subtraction of Fractions

84. 1. Add $\frac{3}{5}$ and $\frac{1}{5}$. (Find the total number of parts.) *Ans.* $\frac{4}{5}$.

2. From $\frac{7}{8}$ subtract $\frac{5}{8}$. (Find the difference in the number of parts.) *Ans.* $\frac{2}{8}$ or $\frac{1}{4}$.

3. Add $\frac{1}{2}$ and $\frac{1}{3}$. (As these fractions are not similar, they have to be changed to similar fractions before they can be added.)

$\frac{1}{2} = \frac{3}{6}$
$+\frac{1}{3} = +\frac{2}{6}$
sum $= \frac{5}{6}$

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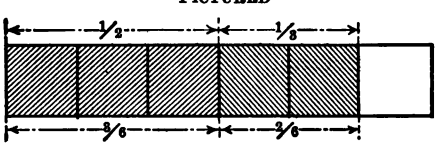
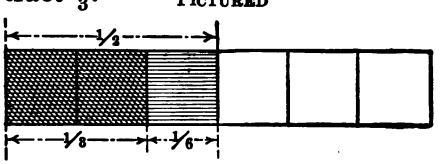


4. From $\frac{1}{2}$ subtract $\frac{1}{3}$.

$\frac{1}{2} = \frac{3}{6}$
$-\frac{1}{3} = -\frac{2}{6}$
difference $= \frac{1}{6}$

PICTURED



5. Add $\frac{1}{2}$ and $\frac{1}{8}$. (Change to 8ths.)

6. From $\frac{1}{2}$ subtract $\frac{1}{8}$.

7. Add $\frac{1}{3}$ and $\frac{1}{4}$. (Change to 12ths.)

8. From $\frac{1}{3}$ subtract $\frac{1}{4}$.

9. Add $\frac{1}{4}$ and $\frac{1}{6}$.

10. Add $\frac{3}{4}$ and $\frac{5}{6}$.

$$\begin{aligned} \tfrac{3}{4} &= \tfrac{9}{12} \\ +\tfrac{5}{6} &= \tfrac{10}{12} \\ \tfrac{19}{12} &= 1\tfrac{7}{12} \end{aligned}$$ *Ans.*

11. From $\frac{3}{4}$ subtract $\frac{2}{3}$.

$$\begin{aligned} \tfrac{3}{4} &= \tfrac{9}{12} \\ -\tfrac{2}{3} &= \tfrac{8}{12} \\ \tfrac{1}{12} & \end{aligned}$$ *Ans.*

12. Add $\frac{7}{8}$ and $\frac{3}{4}$.

13. From $\frac{5}{6}$ subtract $\frac{2}{3}$.

14. Add $\frac{5}{8}$ and $\frac{3}{4}$.

15. From $\frac{3}{4}$ subtract $\frac{5}{8}$.

16. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, and $\frac{1}{4}$.

Convenient form for written work:

$$\begin{aligned} &\text{8ths} \\ \tfrac{1}{2} &= 4 \\ \tfrac{3}{4} &= 6 \\ \tfrac{3}{8} &= 3 \\ +\tfrac{1}{4} &= 2 \\ \tfrac{15}{8} &= 1\tfrac{7}{8} \end{aligned}$$ *Ans.*

17. Add $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$.

18. Add $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{8}$.

19. Add $\frac{2}{3}$, $\frac{5}{9}$, $\frac{5}{6}$, $\frac{13}{18}$.

20. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$.

ADDITION AND SUBTRACTION OF MIXED NUMBERS

85. 1. Add $16\frac{1}{2}$ and $24\frac{1}{3}$. (Add the whole numbers and fractions separately.) *Ans.* $40\frac{5}{6}$.

2. Add $18\frac{1}{2}$, $13\frac{1}{3}$, $14\frac{1}{4}$, $12\frac{5}{6}$.

Convenient form for written work:

	12ths
$18\frac{1}{2}$	6
$13\frac{1}{3}$	4
$14\frac{1}{4}$	3
$+12\frac{5}{6}$	10
$58\frac{11}{12}$	$\frac{23}{12} = 1\frac{11}{12}$.

Ans. $58\frac{11}{12}$.

Note. The use of the line, instead of the equation sign, saves the necessity of writing the entire number a second time. If we consider the line as separating the computations, and think of the work at the right of the line as convenient "side work," we may use the form given. If thought best, the pupils may use the equation sign at first, writing the entire number at the right of the sign, thus: $18\frac{1}{2} = 18\frac{6}{12}$, etc.

3. Add $16\frac{2}{3}$, $18\frac{1}{2}$, $120\frac{3}{4}$, $16\frac{5}{6}$.

4. Add $124\frac{3}{4}$, $130\frac{5}{6}$, $12\frac{1}{2}$, $18\frac{2}{3}$.

5. Add $64\frac{1}{8}$, $32\frac{5}{6}$, $16\frac{1}{2}$, $8\frac{7}{12}$. (Change fractions to 24ths.)

6. Add $\frac{5}{6}$, $18\frac{7}{8}$, $24\frac{7}{16}$. (Change fractions to 48ths.)

Addition of Fractions involving the Use of Least Common Multiple in Finding the Least Common Denominator

86. When the least common denominator cannot be readily seen, we may find it by the process of find-

ing the least common multiple of the denominators, as shown below.

1. Add $5\frac{7}{8}$, $6\frac{5}{12}$, $8\frac{7}{20}$.

	120ths		2)8, 12, 20
$5\frac{7}{8}$	105		2)4 6 10
$6\frac{5}{12}$	50		2 3 5
$+8\frac{7}{20}$	42		$2 \times 2 \times 2 \times 3 \times 5 = 120$
19	197	$\frac{197}{120} = 1\frac{77}{120}$	15 10 6
$1\frac{77}{120}$			8)120 12)120 20)120
$20\frac{77}{120}$ *Ans.*			$15 \times 7 = 105$. $10 \times 5 = 50$. $6 \times 7 = 42$.

EXPLANATION. As "side work" we arrange the denominators as shown, and divide them by a common prime factor, and continue to divide by prime factors as long as two or more of the numbers may be exactly divided by any prime factor. If any one of the numbers cannot be divided by this factor, we bring that number down, dividing the other numbers. When such division has been extended as far as possible, the product of the factors used as divisors and of the remaining numbers will be the least common multiple. Dividing this least common multiple by each denominator, and multiplying the terms of the fraction by the quotient thus obtained, we change the fractions to equivalent similar fractions, which may be added. As the sum is more than one whole unit, we change fraction $\frac{197}{120}$ to mixed number $1\frac{77}{120}$.

2. Add $\frac{11}{12}$, $\frac{7}{15}$, $\frac{5}{18}$.

	180ths
$\frac{11}{12}=$	165
$\frac{7}{15}=$	84
$+\frac{5}{18}=$	50
$1\frac{119}{180}$	299

$3)12,\ 15,\ 18$
$2)\ 4\quad 5\quad 6$
$\quad 2\quad 5\quad 3$

$3\times 2\times 2\times 5\times 3=180$ least common denominator.

$\frac{299}{180}=1\frac{119}{180}$ *Ans.*

3. Add $\frac{4}{21}$, $\frac{11}{12}$, $\frac{13}{14}$.

	84ths
$\frac{4}{21}=$	16
$\frac{11}{12}=$	77
$+\frac{13}{14}=$	78
$2\frac{3}{84}$	171

$7)21,\ 12,\ 14$
$3)\ 3\quad 12\quad 2$
$2)\ 1\quad 4\quad 2$
$\quad 1\quad 2\quad 1$

$\frac{171}{84}=2\frac{3}{84}$ *Ans.*

$7\times 3\times 2\times 2=84$ L. C. D.

4. Add $72\frac{4}{9}$, $12\frac{13}{24}$, $7\frac{23}{36}$.

5. Add $60\frac{3}{8}$, $\frac{11}{36}$, $12\frac{11}{24}$.

6. Add $62\frac{5}{12}$, $18\frac{7}{16}$, $24\frac{19}{24}$.

7. Add $15\frac{3}{7}$, $12\frac{3}{4}$, $18\frac{1}{2}$, $45\frac{5}{14}$.

8. Add $3\frac{4}{5}$, $16\frac{7}{10}$, $42\frac{8}{25}$, $32\frac{3}{5}$.

9. Add $32\frac{2}{3}$, $15\frac{5}{7}$, $8\frac{4}{9}$, $12\frac{17}{21}$.

10. Add $5\frac{2}{3}$, $14\frac{3}{5}$, $6\frac{4}{21}$, $8\frac{6}{35}$, $5\frac{11}{42}$.

11. Add $6\frac{7}{8}$, $18\frac{3}{4}$, $24\frac{9}{16}$, $42\frac{23}{48}$.

12. Add $5\frac{14}{15}$, $8\frac{3}{5}$, $9\frac{13}{25}$.

13. Add $5\frac{7}{9}$, $8\frac{8}{15}$, $9\frac{7}{24}$, $6\frac{1}{4}$.

Note. In business we seldom use fractions with such large denominators that we cannot easily determine the least common denominator without the use of the process of finding the least common multiple.

SUBTRACTION OF MIXED NUMBERS

Subtraction of Mixed Numbers involving Reduction of a Unit in the Minuend

87. 1. From $18\frac{3}{4}$ subtract $12\frac{1}{2}$. *Ans.* $6\frac{1}{4}$.

	4ths
$18\frac{3}{4}$	3
$-\ 12\frac{1}{2}$	2
$6\frac{1}{4}$	1

2. From $18\frac{2}{8}$ subtract $12\frac{1}{2}$.

3. From $25\frac{7}{8}$ subtract $12\frac{3}{4}$.

4. From $16\frac{1}{2}$ subtract $4\frac{3}{4}$. *Ans.* $11\frac{3}{4}$.

	4ths
$16\frac{1}{2}$	2
$-\ 4\frac{3}{4}$	3
$11\frac{3}{4}$	3

Think, $-\frac{3}{4}$ and what are $\frac{6}{4}$? 5 and what are 6? 0 and what are 1? The one unit which we reduce in the minuend is equal to $\frac{4}{4}$, which united with the given $\frac{2}{4}$ makes $\frac{6}{4}$ for the fraction of the minuend.

5. From $24\frac{1}{8}$ subtract $16\frac{1}{2}$.

	8ths
$24\frac{1}{8}$	1
$-\ 16\frac{1}{2}$	4
$7\frac{5}{8}$	5

Think, $-\frac{4}{8}$ and what are $\frac{9}{8}$; 17 and what are 24?

Note. In subtraction of integers, as in $\begin{array}{r} 34 \\ -\ 15 \\ \hline \end{array}$, we think, 5 and what are 14, as the 4 is tenths of the next number, and 10 and 4 are 14. In subtracting in the 5th example given above, we think, 4 and what are 9, as the 1 is 8ths of the next number, and 8 and 1 are 9. The form of arrangement shown expresses the parts of the next number in a very convenient way, and suggests the guide to the reduction of the unit.

6. From $16\frac{2}{3}$ subtract $4\frac{3}{4}$.

7. From $185\frac{3}{4}$ subtract $67\frac{7}{8}$.

8. From $120\frac{4}{5}$ subtract $56\frac{5}{6}$.

9. From $18\frac{1}{3}$ subtract $12\frac{7}{8}$.

10. From $75\frac{7}{9}$ subtract $62\frac{9}{10}$.

11. From 250 subtract $65\frac{4}{7}$.

12. From the sum of $16\frac{1}{4}$ and $12\frac{1}{8}$, subtract $25\frac{5}{6}$.

13. A man owned 150 acres of land. He sold $65\frac{3}{4}$ acres to one man and $16\frac{5}{8}$ acres to another man. How many acres had he left?

14. From a bin containing 75 bushels of grain, $24\frac{3}{4}$ bushels were sold and $16\frac{3}{8}$ bushels used for seed. How many bushels remained in the bin?

15. From a pole $28\frac{1}{2}$ ft. long a piece $14\frac{3}{4}$ ft. long was cut. How long was the part of the pole that remained?

16. The distance between two places is $150\frac{1}{4}$ miles. What distance remains to go, after one has traveled $130\frac{7}{8}$ miles of the route?

17. After selling $3\frac{5}{8}$ acres of land, a farmer had $123\frac{3}{4}$ acres. How much more must he buy to have 150 acres?

18. Mary is $12\frac{5}{12}$ years old and her sister Sarah is $9\frac{2}{3}$ years old. How much older is one sister than the other?

19. A pail filled with candy weighs $14\frac{1}{8}$ pounds. When empty, the pail weighs $2\frac{7}{8}$ pounds. How many pounds of candy were there in the pail?

MULTIPLICATION AND DIVISION OF FRACTIONS

88. Multiplication of Fractions

1. Multiply $\frac{1}{3}$ by 2.

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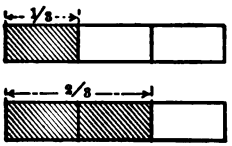
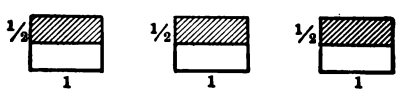


$$\frac{1}{3} \times 2 = \frac{1 \times 2}{3 \times 1} = \frac{2}{3}. \quad \textit{Ans.}$$

2. Multiply 3 by $\frac{1}{2}$.

$$3 \times \frac{1}{2} = \frac{3 \times 1}{1 \times 2} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \textit{Ans.}$$

PICTURED



3. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}. \quad \textit{Ans.}$$

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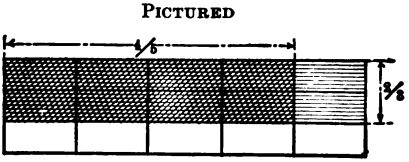


4. Multiply $3\frac{1}{2}$ by $2\frac{1}{4}$.

$3\frac{1}{2} = \frac{7}{2}$
$2\frac{1}{4} = \frac{9}{4}$

$$\frac{7}{2} \times \frac{9}{4} = \frac{7 \times 9}{2 \times 4} = \frac{63}{8} \text{ or } 7\tfrac{7}{8}. \quad \textit{Ans.}$$

89. PRINCIPLE. In the multiplication of fractions, the product of the numerators gives the number of parts, and the product of the denominators gives the size of the parts of the required result.

5. Multiply $16\frac{2}{3}$ by $3\frac{1}{4}$.

6. Multiply $\frac{8}{9}$ by $\frac{4}{5}$.

7. Multiply $\frac{12}{25}$ by $\frac{5}{8}$.

$$\frac{12}{35} \times \frac{5}{8} = \frac{\cancel{12}^{3} \times \cancel{5}^{1}}{\cancel{35}_{5} \times \cancel{8}_{2}} = \frac{3}{10}. \quad \textit{Ans.}$$

The operations are shortened by cancellation. The factors 4 and 5 are canceled from both numerator and denominator. This does not change the value of the product, but it shortens the work. Cancellation should be used as often as possible.

8. Multiply $\frac{12}{35}$ by $\frac{7}{8}$.

9. Multiply $\frac{16}{25}$ by $\frac{5}{8}$.

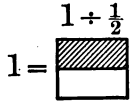
10. Multiply $\frac{24}{35}$ by $\frac{5}{6}$.

Division of Fractions

90. 1. Divide 3 by $\frac{1}{2}$.

$$3 \div \frac{1}{2} = \frac{3 \times 2}{1 \times 1} = 6. \quad \textit{Ans.}$$

$1 \div \frac{1}{2}$



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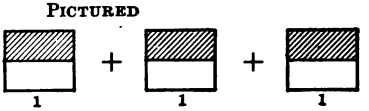

2. Divide $\frac{1}{2}$ by 3.

$$\frac{1}{2} \div 3 = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}. \quad \textit{Ans.}$$

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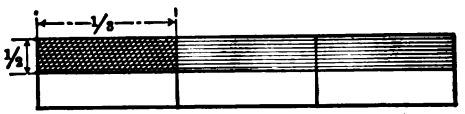

3. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

$$\frac{2}{3} \div \frac{3}{4} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}. \quad \textit{Ans.}$$

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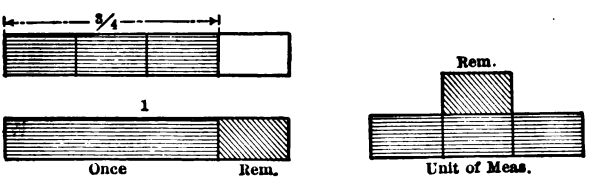


$1 \div \frac{3}{4} = 1\frac{1}{3}$ or $\frac{4}{3}$.

$\frac{2}{3}$ of $1 \div \frac{3}{4} = \frac{2}{3}$ of $\frac{4}{3}$ or $\frac{8}{9}$. *Ans.*

4. Divide $\frac{3}{4}$ by $\frac{7}{8}$. *Ans.* $\frac{6}{7}$.

Analysis: 1 contains $\frac{7}{8}$, $\frac{8}{7}$ times.

$\frac{3}{4}$ of 1 contains $\frac{7}{8}$, $\frac{3}{4}$ of $\frac{8}{7}$ times, or $\frac{6}{7}$. *Ans.*

Written form: $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$.

Note. If further study of the principle of inverting the divisor in division of fractions be desired, the facts are very clearly shown by use of folded paper in measuring. To prove that $\frac{3}{4} \div \frac{7}{8} = {}^{8} \times \frac{3}{7}$, cut two pieces of paper of the same size, one to represent the unit and the other folded to represent $\frac{7}{8}$ of the unit.

Use the $\frac{1}{4}$ piece as the unit of measure, and try how many times the whole unit contains it. It will be found that the whole unit contains the $\frac{1}{4}$ piece once and leaves a remainder which is $\frac{1}{4}$ of the unit of measure. Therefore, it contains it $1\frac{1}{4}$ times or $\frac{5}{4}$ times. Any number of times the unit will, therefore, contain the $\frac{1}{4}$ piece $\frac{5}{4}$ of that number of times.

91. To divide by a fraction, multiply by that fraction inverted.

Exercises in Multiplication and Division of Fractions

92. 1. Multiply $\frac{2}{3}$ by $\frac{3}{4}$ and divide the product by $\frac{3}{8}$.

2. Divide $\frac{6}{7}$ by $\frac{2}{3}$ and multiply the quotient by $\frac{1}{2}$.

3. Multiply $\frac{5}{6}$ by $\frac{2}{3}$ and divide the product by $\frac{4}{5}$.

4. Divide 3 by $\frac{1}{4}$ and multiply the quotient by 8.

5. Divide $\frac{6}{7}$ by $\frac{3}{9}$ and multiply the quotient by $2\frac{1}{2}$.

6. Divide $3\frac{1}{2}$ by 4 and multiply the quotient by $\frac{7}{8}$.

7. Divide 6 by $\frac{1}{3}$ and multiply the quotient by $\frac{1}{6}$.

93. Reduce the following fractional expressions to their simplest form by dividing the numerators by the denominators:

1. $\frac{\frac{3}{4}}{\frac{2}{3}}$. $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$. *Ans.*

2. $\frac{\frac{2}{3} \times \frac{3}{4}}{\frac{1}{2} + \frac{1}{3}}$. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ simple numerator
$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ simple denominator
$\frac{1}{2} \div \frac{5}{6} = \frac{3}{5}$ value of the fraction. *Ans.*

3. Find the value of $\frac{3\frac{1}{2}}{4\frac{3}{8}}$.

$3\frac{1}{2} = \frac{7}{2}$ simple numerator
$4\frac{3}{8} = \frac{35}{8}$ simple denominator
$\frac{7}{2} \div \frac{35}{8} =$ —— *Ans.*

4. Find the value of $\frac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2} \times \frac{1}{4}}$.

5. Find the value of $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{3}{4} - \frac{1}{2}}$.

6. Find the value of $\frac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$.

7. Find the value of $\frac{6\frac{2}{3} + 3\frac{1}{8}}{\frac{4}{5} + 3\frac{5}{6}}$.

8. Find the value of $\frac{\frac{6}{7} + \frac{2}{3}}{\frac{4}{5} \text{ of } \frac{8}{9}}$.

MIXED NUMBERS WITH LARGE INTEGRAL PARTS IN MULTIPLICATION AND DIVISION

Convenient Form for Computation in Multiplication

94. 1. Multiply $1846\frac{2}{3}$ by 12.

$$
\begin{array}{rl}
1846\frac{2}{3} & \\
\times\ 12 & \\
\hline
8 & (12 \text{ times } \frac{2}{3}) \\
3692 & (2 \text{ times } 1846) \\
1846 & (10 \text{ times } 1846) \\
\hline
22160 & (12\frac{2}{3} \text{ times } 1846)
\end{array}
$$

2. Multiply 1473 by $8\frac{1}{2}$.

$$\begin{array}{r} 1473 \\ \times\ 8\frac{1}{2} \\ \hline 736\frac{1}{2} \quad (\frac{1}{2} \text{ of } 1473) \\ 11784 \quad (8 \text{ times } 1473) \\ \hline 12520\frac{1}{2} \quad (8\frac{1}{2} \text{ times } 1473) \end{array}$$

3. Multiply 3468 by $7\frac{3}{4}$.

$$\begin{array}{r} 3468 \\ \times\ 7\frac{3}{4} \\ \hline 867 \quad (\frac{1}{4} \text{ of } 3468) \\ 2601 \quad (\frac{3}{4} \text{ of } 3468) \\ 24276 \quad (7 \text{ times } 3468) \\ \hline 26877 \quad (7\frac{3}{4} \text{ times } 3468) \end{array}$$

4. Find the cost of $345\frac{7}{8}$ acres of land at \$64 an acre.

$$\begin{array}{rl} 345\frac{7}{8} & \\ 64 & \\ \hline 56 & (64 \times \frac{7}{8}) \\ 1380 & (4 \times 345) \\ 2070 & (60 \times 345) \\ \hline 22136 & (64 \times 345\frac{7}{8}) \end{array}$$

(Decide the terms of the answer, then make the computation in the abstract form.)

\$—— *Ans.*

\$ 22136 *Ans.*

5. Find the cost of $1250\frac{3}{8}$ bushels of oats at \$.32 a bushel.

6. Find the cost of $84\frac{3}{4}$ yards of cloth at \$.48 a yard.

7. Find the cost of $120\frac{3}{4}$ acres of land at \$ 96 per acre.

8. A man worked $18\frac{2}{3}$ days at \$4.50 per day. How much did he earn?

9. Find the cost of $150\frac{3}{4}$ yards of cloth at \$.64 a yard.

10. Find the cost of $2842\frac{7}{8}$ bushels of wheat at \$.96 a bushel.

Convenient Form for Computation in Division

95. 1. Divide $4826\frac{1}{2}$ by 8.

$$8\overline{)4826\frac{1}{2}}\quad 603\tfrac{5}{16}$$

Divide as far as possible in the usual way. When the remainder $2\frac{1}{2}$ is found, change that to an improper fraction, $\frac{5}{2}$, and divide it by 8. $\frac{5}{2} \div 8 = \frac{5}{16}$.

2. Divide $7264\frac{3}{8}$ by 4.

3. Divide $3692\frac{4}{5}$ by 3.

4. Divide $7263\frac{7}{12}$ by 3.

5. Divide $8645\frac{2}{3}$ by 9.

6. Divide $7130\frac{5}{8}$ by 8.

7. Divide $1234\frac{6}{7}$ by 6.

8. Divide $760\frac{1}{3}$ by 24.

$$24\overline{)760\frac{1}{3}}\quad 31\tfrac{4\,9}{7\,2}$$

$$\begin{array}{r} 72 \\ \hline 40 \\ 24 \\ \hline 16\frac{1}{3} = \frac{49}{3}. \end{array}$$

$\frac{49}{3} \div 24 = \frac{49}{72}$.

Second Plan.
$$24\overline{)760\tfrac{1}{3}} \qquad 31\tfrac{49}{72}$$
$$72\overline{)2281} \qquad \text{(Changing both to thirds.)}$$
$$\begin{array}{r} 216 \\ \hline 121 \\ 72 \\ \hline 49 \end{array}$$

9. Divide $5156\frac{2}{3}$ by 48.
10. Divide $675\frac{1}{2}$ by 25.
11. Divide $1769\frac{3}{4}$ by 36.
12. Divide $3430\frac{7}{8}$ by 48.
13. Divide $76325\frac{4}{5}$ by 15.
14. Divide $67894\frac{3}{4}$ by 64.
15. Divide $621045\frac{3}{8}$ by 24.

FINDING WHAT FRACTIONAL PART ONE NUMBER IS OF ANOTHER

Introduction

96. 1. One is what part of two?
2. 4 is what part of 12?
3. What part of 12 is 4?
4. What part of \$2 is \$1.50?
5. What part of 1 foot is 6 inches?
6. 9 inches is what part of 1 foot?
7. 2 months is what part of 1 year?

8. What part of 1 year is 3 months?

9. What part of 1 gallon is 3 quarts?

10. 2 feet is what part of 1 yard?

97. To find what fractional part one number is of another: Take the number which denotes the part for the numerator and the number which denotes the whole for the denominator.

Exercises in finding Fractional Parts

98. 1. A football team played 12 games and lost 3. What part of their games did they lose? What part did they win?

2. A man had \$ 50 and spent \$ 8 for a coat. What part of his money did he spend? What part did he have left?

3. From a barrel containing $31\frac{1}{2}$ gallons of oil $6\frac{1}{2}$ gallons are taken. What part of the whole remained?

4. A man had 1250 acres of land and sold 350 acres. What part of his land had he left?

5. A man had 750 sheep and sold 240 of them. What part of his flock did he sell?

6. 350 is what part of 900?

7. What part of 750 is 200?

8. 165 is what part of 660?

9. What part of 10000 is 875?

10. 125 is what part of 2000?

FINDING THE WHOLE FROM A FRACTIONAL PART

99. 1. If $\frac{3}{8}$ of an acre of land is worth \$ 36, what is 1 acre worth?

$\frac{3}{8}$ acres are worth \$ 36.

1 acre is worth \$ 36 ÷ $\frac{3}{8}$ or \$ 96. *Ans.*

Note. The "old line" analysis and solution for this class of examples may be used if it be preferred. It was as follows:

If $\frac{3}{8}$ acres are worth \$36,

$\frac{1}{8}$ acre is worth $\frac{1}{3}$ of \$36, or \$12.

$\frac{8}{8}$ or 1 acre is worth 8 times \$12 or \$96.

This form is often confusing to the child, especially in oral analysis, if he give it in full and states that "$\frac{1}{3}$ is worth $\frac{1}{3}$," etc., as a part of the analysis.

If the fraction $\frac{3}{8}$ is considered as an entire expression of quantity, as 12, 15, or 18 would be considered if used in its place, the analysis becomes very simple, establishes a type form for general application, and the child is less likely to make errors in its application. If 3 acres cost \$36, what will 1 acre cost? If $\frac{3}{8}$ acres cost \$36, what will 1 acre cost? If 2 acres cost \$36, what will 1 acre cost? If .25 acres cost \$36, what will 1 acre cost? Exercises like those just given may all be placed under the general type of "dividing the given cost by the given quantity to find the cost of 1."

2. Paid 60¢ for $\frac{4}{5}$ yard of cloth. What would 3 yards cost at that rate?

3. Paid 75¢ for 6 yards of cloth. What will $3\frac{1}{2}$ yards cost at that rate?

4. If $3\frac{1}{2}$ acres of land cost \$ 252, what will $5\frac{1}{4}$ acres cost?

5. If $\frac{2}{3}$ of a farm is worth \$ 1500, what is the value of the whole farm?

6. After losing a part of the money, a man had \$60 or $\frac{3}{4}$ of it left. How much did he lose?

7. If a train runs 36 miles in $\frac{3}{4}$ of an hour, how far will it run in $2\frac{1}{2}$ hours at the same rate?

8. After spending $\frac{3}{5}$ of his money, a man has \$50 left. How much had he at first?

9. In a school $\frac{4}{7}$ of the pupils are girls and there are 360 boys. How many pupils in the school?

10. If $\frac{2}{7}$ yard of cloth cost 15¢, what will 5 yards cost at the same rate?

11. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{4}$ in another, and the remainder, which was 77, in the third pasture. How many sheep had he?

$\frac{1}{5}+\frac{1}{4}=\frac{9}{20}$ part in the first and second pastures.

$\frac{20}{20}-\frac{9}{20}=\frac{11}{20}$ in the third pasture.

$\frac{11}{20}$ of the entire number of sheep $= 77$.

Entire number of sheep $= 77 \div \frac{11}{20} = 140$. *Ans.*

12. A man lost $\frac{1}{2}$ of his money, then earned \$18, when he had $\frac{4}{5}$ as much as he had at first. How much had he at first?

13. A man spends \$1500 a year, which is $\frac{2}{3}$ of his income. How much is his income?

14. At the rate of 3 for 5¢, what will 18 oranges cost?

15. There are 1200 girls in a school and they are $\frac{5}{8}$ of the entire number of pupils. How many pupils in the school?

16. If $\frac{3}{5}$ of the cost of a farm is $1500, what is the value of the entire farm at the same rate?

17. At the rate of 8 for 25¢, what will 1 dozen of eggs cost?

18. Henry spilled $\frac{1}{5}$ of his berries and had 8 quarts left. How many quarts of berries had he at first?

19. If $\frac{3}{8}$ of a ton of hay cost $7.50, what part of a ton can be bought for $15?

20. If $\frac{5}{6}$ of a cord of wood costs $4, what will $2\frac{1}{2}$ cords cost at the same rate?

CHANGING FRACTIONS TO DECIMALS AND DECIMALS TO FRACTIONS

Changing Decimals to Common Fractions

100. 1. Change .5 to a common fraction.

$$.5 = \frac{5}{10} = \frac{1}{2}. \quad \textit{Ans.}$$

2. Change .05 to a common fraction.

$$.05 = \frac{5}{100} = \frac{1}{20}. \quad \textit{Ans.}$$

3. Change .005 to a common fraction.

$$.005 = \frac{5}{1000} = \frac{1}{200}. \quad \textit{Ans.}$$

4. Change .075 to a common fraction.

$$.075 = \frac{75}{1000} = \frac{3}{40}. \quad \textit{Ans.}$$

5. Change 8.075 to fractional form.

$$.075 = \frac{3}{40}.$$
$$8.075 = 8\frac{3}{40}. \quad \textit{Ans.}$$

Reduce the following to their simplest fractional forms:

6. .28125.	10. 4.025.	14. .9375.	18. 5.0008.
7. .0625.	11. 6.875.	15. 4.125.	19. 6.0075.
8. 65.625.	12. .0875.	16. .0325.	20. 15.0015.
9. .6875.	13. .16.	17. 4.1575.	

Changing Common Fractions to Decimal Form

Introduction

101. 1. Reduce $\frac{1}{2}$ to a decimal.

$$2\overline{)1.0}$$

.5 *Ans.*

2. Reduce $\frac{1}{4}$ to a decimal.

$$4\overline{)1.00}$$

.25 *Ans.*

3. Reduce $\frac{3}{8}$ to a decimal.

$$8\overline{)3.000}$$

.375 *Ans.*

102. To change a fraction to a decimal divide the numerator by the denominator.

103. Change the following fractions to decimal form:

1. $\frac{5}{8}$.	5. $\frac{15}{16}$.	9. $\frac{25}{36}$.	13. $6\frac{5}{16}$.	17. $14\frac{3}{20}$.
2. $\frac{7}{8}$.	6. $\frac{8}{15}$.	10. $\frac{9}{25}$.	14. $7\frac{5}{12}$.	18. $7\frac{21}{80}$.
3. $\frac{5}{16}$.	7. $\frac{5}{12}$.	11. $\frac{36}{75}$.	15. $14\frac{3}{8}$.	19. $25\frac{8}{11}$.
4. $\frac{7}{16}$.	8. $\frac{5}{64}$.	12. $4\frac{7}{8}$.	16. $3\frac{7}{40}$.	20. $10\frac{3}{25}$.

21. $4\frac{2}{3}$. 23. $\frac{8}{9}$. 25. $\frac{27}{35}$. 26. $\frac{18}{45}$. 27. $\frac{6}{7}$.

22. $\frac{5}{6}$. 24. $\frac{11}{15}$.

104. Divide as indicated, and carry out the remainder to three decimals:

1. $7 \div 27$.
2. $5 \div 6$.
3. $674 \div 36.34$.
4. $27.3782 \div 4.3267$
5. $8.47326 \div 75.43$.
6. $.8487564 \div .075637$.
7. $4273 \div 5737$.
8. $1 \div 1.007633$.
9. $8972.436 \div 756.3452$.
10. $8 \div 9$.

SUMMARY OF FRACTIONS

105. 1. A fraction is one or more of the equal parts of a unit.

2. The numerator of a fraction expresses the number of parts taken.

3. The denominator shows into how many parts the unit has been divided. It, therefore, expresses the size of the parts.

4. Reduction is the process of changing the form without changing the value.

5. To reduce fractions to their lowest terms, we reject common factors from both numerator and denominator.

6. An improper fraction has a numerator greater than its denominator.

7. To change an improper fraction to an integer, mixed number, or decimal, we divide its numerator by its denominator.

8. A mixed number is a whole number and a fraction taken together.

9. To change a mixed number to an improper fraction, we change the whole number to the number of parts shown by the denominator of the fraction, then unite these parts with the fraction.

10. Similar fractions have like denominators.

11. Fractions must be made similar before they can be added or subtracted.

12. To add or subtract fractions, we change them to similar fractions, then find the sum or difference of their numerators, and write the result as the numerator of the fraction which has the common denominator.

13. To add or subtract mixed numbers, we add or subtract the fractional and integral parts separately.

14. When the least common denominator cannot be easily found without the process, we find the least common multiple of the denominators for the least common denominator.

15. To multiply fractions, we multiply the numerators together for the numerator of their product, and the denominators together for the denominator of their product.

16. In the multiplication of fractions we may often shorten the process by canceling equal factors from the numerators and the denominators. This process, called cancellation, does not change the value of the product.

17. To multiply an integer by a fraction, or a fraction by an integer, we may express the integer in fractional form, and apply the general rule for multiplication of fractions.

18. To divide by a fraction, we multiply by that fraction inverted.

19. In division of fractions, we may express the integers in fractional form, and apply the general rule.

20. To find the value of any fractional expression we divide the numerator by the denominator.

21. In using mixed numbers with large integral parts in multiplication, we may multiply the whole number and the fractional part separately, and then unite their products.

22. In dividing a mixed number with a large integral part by an integer, we divide the integral part as far as possible, then change the remainder, including the fractional part, to an improper fraction and complete the division.

23. To find what fractional part one number is of another, we express the number denoting the

part as the numerator, and the number denoting the whole as the denominator of a fraction.

24. To find the value of the whole unit when the value of a part is given, we divide the given value by the given part.

25. To change decimals to fractional form, we supply the indicated denominator and reduce the resulting fraction to its simplest form.

26. To change fractions to decimal form, we divide the numerator by the denominator.

27. Fractions indicate division. The value of a fraction is the quotient obtained by dividing the numerator by the denominator.

28. Both numerator and denominator of a fraction may be either multiplied or divided by the same number without changing the value of the fraction.

29. Multiplying the numerator of a fraction multiplies its value. Dividing the numerator of a fraction divides its value.

30. Multiplying the denominator of a fraction divides the value. Dividing the denominator of a fraction multiplies its value.

31. Any change in the numerator produces a like change in the value of the fraction.

32. Any change in the denominator produces an opposite change in the value of the fraction.

Solution of Problems

106. 1. Make an abstract or use a diagram to show clearly what is given and what is required in the problem.

2. Determine the general question of the problem, and the terms in which the answer must be expressed, that you may make the computations in abstract form.

3. Decide upon the steps of the solution, and indicate these steps clearly in the outline of your plan of solution. In the work of solution name clearly the results obtained by each step.

4. Perform the operations in the easiest way, the terms being arranged in the most convenient form for computation. Answer the questions step by step, giving full consideration to each one in order, as if it were a problem by itself.

5. Check or prove your answers.

6. It often helps, in making the outline, to substitute easy and simple numbers for the large or complex ones given, keeping the general questions the same as those of the given problem.

Miscellaneous Problems for General Review

107. 1. If $\frac{3}{4}$ of 9 bushels of wheat cost $\$13\frac{1}{2}$, what will $\frac{7}{8}$ bushel cost?

2. If $7\frac{1}{2}$ tons of hay cost $\$120$, how many tons can be bought for $\$78$?

3. If a man travel 240 miles in $5\frac{3}{4}$ days, how far would he travel in $3\frac{1}{2}$ days at the same rate?

4. If the dividend is $\frac{7}{9}$ and the quotient $\frac{4}{3\,9}$, what is the divisor?

5. A can do a piece of work in 8 days, B can do it in 6 days. How long will it take them to do the work, working together?

A does $\frac{1}{8}$ of it each day.

B does $\frac{1}{6}$ of it each day.

A and B could do $\frac{1}{8}+\frac{1}{6}$ of it each day, or $\frac{7}{24}$.

$\frac{24}{24} \div \frac{7}{24} = 3\frac{3}{7}$, the number of days required.

6. A man raised $93\frac{3}{4}$ bushels of oats on $3\frac{7}{8}$ acres. How many bushels an acre did he raise?

7. A train travels $\frac{3}{7}$ of a mile a minute. What is its rate of speed per hour?

8. A man's property is \$ 5000 and he owes \$ 6000. What part of his debts can he pay? How much could he pay a man to whom he owed \$ 500, if he paid him his regular share?

9. A man can plaster a house in 12 days. What part of it can he plaster in $3\frac{1}{2}$ days?

10. If a man can do $\frac{3}{8}$ of a piece of work in 9 days, how many days will it take him to do the entire work?

11. If a man does $\frac{2}{7}$ of a piece of work in 4 days, how long will it take him to complete it?

12. If a cup holds $\frac{1}{4}$ of a quart, how many cupfuls are there in $11\frac{7}{9}$ quarts of water?

13. Grain costing 48¢ a bushel is sold for 60¢ a bushel. What part of the cost is gained?

14. How much is lost by selling a farm which cost $3000 for $\frac{4}{5}$ of its cost?

15. John can pick a quart of berries in $\frac{2}{3}$ of an hour. How many quarts can he pick in 2.5 hours?

16. James sold his bicycle for $\frac{2}{3}$ of its cost, losing $10. What was the cost?

17. Which is the greater and how much, 1.875 or $1\frac{11}{12}$?

18. A bale of cotton sold at the rate of $9\frac{2}{5}$¢ a pound brought $33.84. How many pounds were there in the bale?

19. If .125 of an acre of land is worth $15, what are $3\frac{1}{4}$ acres worth?

20. I buy apples at the rate of 10 for 3¢ and sell them at the rate of 6 for 5¢. How much do I make on 30 apples?

21. A boy buys peaches at the rate of 5 for 2¢ and sells them at the rate of 4 for 3¢. How many must he buy and sell to make a profit of $4.20?

22. A train runs $\frac{3}{4}$ of a mile in $\frac{4}{5}$ of a minute. What is its rate of speed per hour?

23. Reduce to its simplest form $\frac{\frac{4}{5} \text{ of } \frac{1}{3}}{8\frac{1}{4}}$.

24. After buying a suit of clothes for $35, I find that I have $\frac{3}{8}$ of my money left. How much money had I at first?

25. What part of $\frac{2}{3}$ is $\frac{1}{2}$?

26. A can do a piece of work in 5 days, B can do it in 8 days. How long will it take them if they work together?

27. Which is the greater, and how much, $\frac{39}{104}$ or $\frac{5}{13}$?

28. Add $4\frac{1}{2}$, 6.125, $13\frac{5}{8}$, 12.875, $4\frac{3}{4}$.

29. From $17\frac{7}{8}$ subtract 4.325.

30. A farmer raised 639 bushels of potatoes and sold 497 bushels. What part of his crop had he left?

31. The difference between $\frac{1}{2}$ of a number and $\frac{1}{3}$ of it is 17. What is the number?

32. I paid \$250 for a horse and had $\frac{3}{5}$ of my money left. How much money had I left?

33. Divide .0024 by $\frac{3}{8}$.

34. From $14\frac{7}{16}$ pounds of butter, $5\frac{3}{4}$ pounds were sold to one person and $3\frac{7}{8}$ pounds to another. How many pounds remained?

35. A man sold a horse for $\frac{3}{4}$ of its cost, losing \$50. What did the horse cost?

36. How many pieces of wire, each $\frac{2}{3}$ yard in length, can be cut from a wire 4 yards long?

37. There are 32 quarts in one bushel. What decimal of a bushel is 3 quarts?

38. What will $\frac{2}{3}$ of a yard of cloth cost if .8 of a yard cost \$1.60?

39. A man owns $\frac{2}{5}$ of a house and sold $\frac{3}{4}$ of his share for $1500. What is the value of the entire house at that rate?

40. If $\frac{5}{8}$ of a cord of wood is worth $3.75, what is .75 of a cord worth?

41. $12\frac{3}{7}$ is what part of 29?

42. What fraction added to $\frac{2}{7}$ gives $\frac{3}{4}$?

43. If $\frac{3}{8}$ yard of cloth cost 18¢, what will 4.875 yards cost?

44. Find the cost of $1438\frac{2}{3}$ bushels of grain at 84¢ a bushel.

45. In 30 loads of hay there were $48\frac{1}{2}$ tons. What was the average weight of each load in tons? In pounds? (2000 pounds to the ton.)

46. What fractional part of a yard is 6 inches?

47. If the average length of the step of a boy is $\frac{2}{3}$ yard, how many steps will he take in walking $\frac{1}{4}$ mile? (1760 yards = 1 mile.)

48. If the average length of a step of a boy is $\frac{2}{3}$ yard, how far will he walk in taking 1000 such steps?

49. A tank when $\frac{2}{3}$ full contains 48 gallons. How much will it contain when $\frac{3}{4}$ full?

50. If $\frac{3}{8}$ interest in a stock company costs $12,000, what should $\frac{2}{5}$ interest cost?

51. If $\frac{7}{8}$ bushel of wheat cost $.98, what will 4.125 bushels cost at the same rate?

52. Sold 48 acres of land for $3840. What should I receive for $12\frac{3}{8}$ acres at the same rate?

53. May is $\frac{2}{3}$ as old as her sister Kate, and the difference in their ages is 6 years. What is the age of each?

54. After working 8 days, a man finds that he has done $\frac{2}{5}$ of his work of digging a trench. How long should it take him to complete the job?

55. A man invested $\frac{1}{2}$ of his money in real estate, $\frac{1}{5}$ of the remainder in wheat, and had $1000 left. How much had he at first?

56. A can do a piece of work in 5 days, B in 6, and C in 8 days. How long will it take them to do the job, working together?

57. A man paid $51 for $3\frac{2}{5}$ tons of hay. What was the price per ton?

58. What will 1975 pounds of hay cost at $24 a ton?

59. A man owned $\frac{5}{8}$ of a farm and sold $\frac{2}{3}$ of his share. What part of the farm did he still own?

60. How many coats can be made from $33\frac{3}{4}$ yards of cloth if $1\frac{7}{8}$ yards are required for each coat?

61. A son gets $\frac{8}{9}$ of an estate and invests $1600, which was $\frac{5}{8}$ of his share. What was the value of the estate?

62. A lady buys 10 yards of ribbon and uses $6\frac{2}{3}$ yards. What part of the ribbon has she left?

63. At $\$\frac{7}{8}$ per yard, how much silk can be bought for \$15.75?

64. If .625 of a barrel of flour is worth \$4, what is $\frac{3}{4}$ of a barrel worth?

65. The sum of two fractions is $\frac{125}{176}$, and the smaller fraction is $\frac{116}{704}$. What is the larger fraction?

CHAPTER VIII

DENOMINATE NUMBERS

108. A concrete number in which the unit of measure has a fixed value is called a denominate number.

4 feet, 30 minutes, 2 pounds, 4 yards, etc., are denominate numbers.

COMPARISON OF OPERATIONS WITH SIMPLE AND DENOMINATE NUMBERS

109. Simple Number	Denominate Number
324 or 3 hundreds, 2 tens, 4 units.	3 bushels, 2 pecks, 4 quarts.

REDUCTION

Reduce to lower denominations:

hundreds	tens	units		bushels	pecks	quarts	
5 (10)	3 (10)	6		5 (4)	3 (8)	6	
5	50	530		5	20	184	
	53	536	*Ans.*		23	190	*Ans.*

Ans. 536 units. *Ans.* 190 quarts.

In simple numbers, the place value of the figures by the Arabic system of notation enables us to write at once the number of units, so that we do not have to make the computations shown above.

In denominate numbers, we are often obliged to make the full operations as shown. These operations are: 4 pecks in each bushel, 5 times 4 pecks in 5 bushels or 20 pecks; 20 pecks and the 3 pecks are 23 pecks. 8 quarts in each peck, 23 times 8 quarts in 23 pecks, or 184 quarts; 184 quarts and the 6 quarts are 190 quarts. 5 bu. 3 pk. 6 qt. = 190 quarts.

Reduce to higher denominations:

536 units.

10)536 6 units
10)53 3 tens
5 hundreds

Ans. 536 units = 5 hundreds, 3 tens, 6 units.

190 quarts.

8)190 6 quarts
4)23 3 pecks
5 bushels

Ans. 190 qt. = 5 bu. 3 pk. 6 qt.

ADDITION

hundreds	tens	units	bushels	pecks	quarts
3 (10)	2 (10)	4	3 (4)	2 (8)	4
7	1	6	7	1	6
5	3	4	5	3	4
6	2	5	6	2	5
21	9	9	23	2	3

Ans. 2199. *Ans.* 23 bu. 2 pk. 3 qt.

In adding simple numbers, we carry 1 for each ten, as it takes 10 of the units of each column to make 1 of the next higher column. The scale is uniform.

In adding denominate numbers, the scale varies

for different measures. In denominate numbers the scale may be written between the columns and used as a guide to carrying. The operations are the same, except that we carry 1 for each 8 quarts in adding quarts, and 1 for each 4 pecks in adding pecks, while in simple numbers we carry 10 each time.

SUBTRACTION

tens	units	tenths
8 (10)	3 (10)	0
5	3	1
2	9	9

Ans. 29.9.

gallons	quarts	pints
8 (4)	3 (2)	0
5	3	1
2	3	1

Ans. 2 gal. 3 qt. 1 pt.

In subtracting the denominate numbers in the exercise shown, we first reduce one of the quarts to pints, making 2 pints; in subtracting the simple numbers, we first reduce one of the units to tenths, making 10 tenths. Trace the remainder of the operations with the simple numbers, and then apply the same plan with the denominate numbers, using the scale written between the columns to guide your reductions.

MULTIPLICATION

units	tenths	hundreds
4 (10)	2 (10)	6
		× 5
21	3	0

Ans. 21.30.

bushels	pecks	quarts
4 (4)	2 (8)	6
		× 5
23	1	6

Ans. 23 bu. 1 pk. 6 qt.

The operations in the denominate number exercise are: 5 times 6 quarts = 30 quarts, which are 3 pecks and 6 quarts. 5 times 2 pecks = 10 pecks, and 3 pecks are 13 pecks, which are 3 bushels and 1 peck. 5 times 4 bushels = 20 bushels, and the 3 bushels carried are 23 bushels. Compare these operations with those of your regular practice in multiplication of simple numbers.

DIVISION

2 tens 6 units 8 tenths	2 bushels 3 pecks 7 quarts
2)5 (10) 3 (10) 6	2)5 (4) 3 (8) 6

The operations in the exercise in simple numbers are: 5 tens are 2 times 2 tens, and 1 ten or 10 units remaining. 10 units and 3 units are 13 units; 13 units are 2 times 6 units and 1 unit or 10 tenths remaining.

10 tenths and 6 tenths are 16 tenths; 16 tenths are 2 times 8 tenths. *Ans.* 26.8.

The operations in the exercise in denominate numbers are: 5 bu. are 2 times 2 bu. and 1 bu. or 4 pk. remaining. 4 pk. and 3 pk. are 7 pk.; 7 pk. are 2 times 3 pk. and 1 pk. or 8 qt. remaining; 8 qt. and 6 qt. are 14 qt. 14 qt. are 2 times 7 qt. 2 bu. 3 pk. 7 qt. *Ans.*

110. The operations of reduction, addition, subtraction, multiplication, and division of denominate numbers are like the corresponding operations with simple numbers, except that in denominate num-

bers the scale varies with the different units of measure, while in simple numbers it is uniformly 10.

TABLES OF DENOMINATE NUMBERS

111. Liquid measure is used in measuring liquids. The standard unit of liquid measure is the gallon, containing 231 cubic inches.

TABLE FOR LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

UNIT EQUIVALENTS

gallons		quarts		pints		gills
				1	=	4
		1	=	2	=	8
1	=	4	=	8	=	32

The barrel of $31\frac{1}{2}$ gal. is used in estimating capacity of tanks, etc.; but in business use the barrel is not a fixed measure.

112. Dry measure is used in measuring fruit, grain, vegetables, etc. The standard unit of dry measure is the bushel, containing 2150.42 cu. in.

TABLE FOR DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

UNIT EQUIVALENTS

bushel	peck	quart	pint
		1 =	2
	1 =	8 =	16
1 =	4 =	32 =	64

In measuring small seeds, grain, sand, etc., the measure is "stricken," that is, made even across the top with the top of the measure used. This is the standard bushel of 2150.42 cu. in., being the same as the British Winchester bushel, and adopted by the United States government, in 1836, as our standard unit for dry measure.

In measuring coarse vegetables, like potatoes, apples, etc., and other large objects, the bushel is heaped, or rounded, on top.

The heaped bushel should contain 2747.7167 cu. in.; but in practice a heaped bushel is estimated to contain $1\frac{1}{4}$ stricken bushels. The bushel now used in Great Britain contains 2218.192 cu. in. and is called the imperial bushel in commerce.

The quart of dry measure is larger than the quart of liquid measure.

2150.42 cu. in. $\div$ 32 = 67.2 cu. in. in dry quarts.

231 cu. in. $\div$ 4 = 57.75 cu. in. in liquid quarts.

The standard units of measure in common use in this country were adopted by the United States government in 1836, and complete sets of all weights and measures were delivered at that time by the United States government to the governor

of each state in the Union. State Sealers of Weights and Measures furnish standard sets of weights and measures to towns and cities. The Metric System of weights and measures is used in international dealings, and in scientific work. It has been legalized for use in the United States, but has not come into general use. It is used by nearly all countries except Great Britain and the United States.

Exercises in Measures of Capacity

113. 1. Reduce 8 gal. 3 qt. 1 pt. to pints.

Convenient form:

8 gal.	(4)	3 qt.	(2)	1 pt.
8		32		70
		35		71

Ans. 71 pt.

2. 6 bu. 3 pk. 2 qt. to quarts.

3. Find the cost at 5¢ a quart of 3 pk. 6 qt. of chestnuts.

4. How many pints in 4 gal. 1 qt.?

5. 3 gal. 3 qt. 1 pt. = —— pints.

6. 4 bu. 5 qt. = —— qt.

7. Change 1235 pints to quarts and gallons.

1235 pints = (1235 ÷ 2) quarts = 617 qt. + 1 pt.

Ans. 1235 pints = 154 gal. 1 qt. 1 pt.

Prove the answer by reducing 154 gal. 1 qt. 1 pt. to pints.

8. Reduce 1620 pints to quarts, pecks, and bushels.

1620 pints = —— bu. —— pk. —— qt. —— pt.

1620 pints = 25 bu. 1 pk. 2 qt. 0 pt. *Ans.*

Convenient form:

2)1620 pt.
8)810 qt. . . . 2
4)101 pk. . . . 1
25 bu.

9. Reduce 83 pints to higher denominations (dry measure).

10. Reduce 15 gal. 1 qt. to pints.

11. Reduce 7 gal. 3 qt. 1 pt. to pints.

12. Reduce 28 bu. 3 pk. 5 qt. to quarts.

13. Reduce 127 pints to gallons, etc.

14. A milk dealer sold 23 gal. 3 qt. 1 pt. of milk in pint bottles. How many bottles did he use?

15. Add 16 bu. 3 pk. 5 qt., 18 bu. 1 pk. 7 qt., 24 bu. 1 pk. 3 qt.

16 bu.	(4) 3 pk.	(8) 5 qt.
18	1	7
24	1	3
59 bu.	2 pk.	7 qt. *Ans.*

16. From 7 gal. 1 qt. subtract 3 gal. 3 qt. 1 pt.

7 gal.	(4) 1 qt.	(2) 0 pt.
3	3	1
3 gal.	1 qt.	1 pt. *Ans.*

17. Add 5 gal. 3 qt. 1 pt., 2 gal. 1 qt., 4 gal. 1 qt. 1 pt., and 3 qt. 1 pt.

18. From 12 bu. 1 pk. 6 qt. subtract 8 bu. 2 pk. 7 qt.

19. Multiply 3 bu. 2 pk. 5 qt. by 6.

3 bu. (4) 2 pk. (8) 5 qt.
× 6
21 bu. 3 pk. 6 qt. *Ans.*

20. Multiply 4 gal. 3 qt. 1 pt. by 8.

21. Multiply 16 bu. 3 pk. 6 qt. by 9.

22. Divide 4 bu. 2 pk. 5 qt. 1 pt. by 2.

2 bu. 1 pk. 2 qt. $1\frac{1}{2}$ pt. *Ans.*
2)4 bu. (4) 2 pk. (8) 5 qt. (2) 1 pt.

23. Divide 28 gal. 2 qt. 1 pt. by 5.

24. Divide 3 gal. 3 qt. 1 pt. by 6.

25. Divide 5 bu. 3 pk. 6 qt. by 8.

26. Divide 132 bu. 1 pk. by 46.

2 bu. 3 pk. 4 qt. *Ans.*
46)132 bu. (4) 1 pk. (8) 0 qt.
92 160 184
40 161 184
138 184
23

27. Divide 8 gal. 2 qt. by 3 gal. 1 qt.

8 gal. 2 qt. = 34 qt.
3 gal. 1 qt. = 13 qt.
34 qt. ÷ 13 qt. = $2\frac{8}{13}$. *Ans.*

28. Divide 56 gal. 3 qt. 1 pt. by 48.

29. Divide 8 bu. 3 pk. 1 qt. by 2 pk. 6 qt.

DENOMINATE FRACTIONS AND DECIMALS

114. 1. Change $\frac{3}{8}$ bu. to lower denominations.

$\frac{3}{8}$ bu. $= \frac{3}{8}$ of 4 pk. $= 1\frac{1}{2}$ pk.
$\frac{1}{2}$ pk. $= \frac{1}{2}$ of 8 qt. $= 4$ qt.
$\frac{3}{8}$ bu. $= 1$ pk. 4 qt. *Ans.*

2. Change $\frac{1}{5}$ pt. to fraction of a gallon.

$\frac{1}{5}$ pt. $= \frac{1}{5}$ of $\frac{1}{2}$ qt. $= \frac{1}{10}$ qt.
$\frac{1}{10}$ qt. $= \frac{1}{10}$ of $\frac{1}{4}$ gal. $= \frac{1}{40}$ gal.
$\frac{1}{5}$ pt. $= \frac{1}{40}$ gal. *Ans.*

3. Change .7 gal. to lower denominations.

.7 gal. $=$ .7 of 4 qt. $= 2.8$ qt.
.8 qt. $=$ .8 of 2 pt. $= 1.6$ pt.
.7 gal. $= 2$ qt. 1.6 pt. *Ans.*

4. Change 25 qt. to decimal of a bushel.

25 qt. $=$ 25 times $\frac{1}{8}$ pk. $= 3.125$ pk.
3.125 pk. $=$ 3.125 times $\frac{1}{4}$ bu. $= .78125$ bu.
25 qt. $= .78125$ bu. *Ans.*

or 1 bu. $= 32$ qt.
25 qt. $= \frac{25}{32}$ bu. $= .78125$ bu.

5. Change 3 pk. 6 qt. to bushels.

3 pk. 6 qt. $= 30$ qt.
30 qt. $= \frac{30}{32}$ bu. $= .9375$ bu. *Ans.*

WEIGHTS AND MEASURES

115. Linear measure is used in measuring lengths and distances.

The standard unit of linear measure is the yard.

TABLE FOR LINEAR MEASURE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
1760 yards = 1 mile (mi.)
$16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 1 mile.

UNIT EQUIVALENTS

Mile		Rod		Yard		Foot		Inch
						1	=	12
				1	=	3	=	36
		1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198
1	=	320	=	1760	=	5280	=	63360

The imperial standard yard of Great Britain, based upon the length of a vibrating pendulum, has been the standard in this country until recently, and is still used in some sections.

In 1893 the United States Government standard yard of $\frac{3600}{3937}$ meters was adopted for official use. The two standards are practically the same.

The unit of measure of the rod is passing away, except for the measurement of large land areas. It is little used in cities, distances being given in miles, yards, and feet.

116. Avoirdupois weight is used for all ordinary purposes of weighing.

The standard unit is the pound, formerly based upon the Troy pound of the mint, but recently fixed for official use as $\frac{1}{2.2046}$ kilograms.

TABLES FOR AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

For comparison with other weights, the pound avoirdupois is fixed as 7000 grains. Troy and apothecaries' pounds each contain 5760 such grains.

The following denominations are sometimes used.

14 lb. = 1 stone.
196 lb. flour = 1 barrel.
100 lb. nails = 1 keg.
200 lb. beef or pork = 1 barrel.

The weight per bushel varies in different states of the Union. In general it is as follows:

Wheat,	60 lb.
Rye,	56 lb.
Corn,	56 lb.
Oats,	32 lb.
Barley,	48 lb.
Corn meal,	50 lb.
Potatoes,	60 lb.
Peas,	60 lb.
Beans,	60 lb.

117. Troy weight is used for weighing gold, silver, and jewels. The unit is the pound.

TABLE FOR TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

In weighing diamonds and pearls the carat is often used as the unit. The carat is equal to 3.168 grains, Troy.

The term "carat" is also used to express the proportion of gold and alloy in gold rings, watch cases, etc. 24 parts is used as the standard, 18 carats meaning $\frac{18}{24}$ gold.

MEASURES OF TIME

118.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 ordinary year (yr.)
366 days	= 1 leap year
100 years	= 1 century

In business computations 30 days are often considered 1 month, and 12 such months or 360 days 1 year.

The common year contains 52 weeks and 1 day.

The exact year contains 365 days 5 hours 48 minutes 49.7 seconds, which is very nearly $365\frac{1}{4}$ days. We allow 365 days for a year, and add one more day for the leap year each fourth year. Further adjustment is made at the end of the century. Centennial years not divisible by 400 are not leap years.

The civil day begins and ends at midnight. The civil year begins January 1 and ends December 31.

"Thirty days have September,
April, June, and November."

The other months, except February, have 31 days. February has 28 days in ordinary years and 29 days in leap years.

119. Angular space and arcs of circles are measured in circular or angular measure. The unit is $\frac{1}{360}$ part of the space about a point on a flat surface, or $\frac{1}{360}$ part of the circumference of a circle. This unit is called a degree.

TABLE OF CIRCULAR OR ANGULAR MEASURE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 quadrant of arc or 1 right angle
360 degrees = 1 circumference

COUNTING TABLE

2 = 1 pair
12 = 1 dozen
20 = 1 score
144 = 1 gross

STATIONERS' TABLE OF MEASURE

24 sheets = 1 quire
20 quires = 1 ream

Paper is generally sold in large quantities by the 100, 500, or 1000 sheets, or by the pound.

Exercises and Applications with Various Tables of Weights and Measures

120. 1. A load of wheat weighing 2430 pounds is worth how much at $1.20 a bushel?

2. In 49 bu. 3 pk. 7 qt. how many quarts?

3. In 3199 pints how many bushels?

4. George Washington was born Feb. 22, 1732, and died Dec. 14, 1799. What was his exact age at his death? *Ans.* 67 yr. 9 mo. 22 da.

Year	Month	Day
1799 (12)	12 (30)	14
− 1732	2	22
67	9	22

5. How long was it from June 26, 1893 to April 8, 1909?

6. Benjamin Franklin was born Jan. 18, 1706, and George Washington was born Feb. 22, 1732. How much older was Franklin than Washington?

7. How much less than a right angle is 75° 15′?

8. What part of a mile is 440 yards?

9. A train runs 60 ft. per second. What is its rate of speed in miles per hour?

10. How much tea in 23 chests, each containing 78 lb. 9 oz.?

11. If 1 pk. 4 qt. of wheat cost 72¢, what is the cost per bushel?

12. Find the sum of 18 lb. 10 oz. 14 pwt. 20 gr., 28 lb. 6 oz. 15 pwt. 15 gr., 36 lb. 4 oz. 12 pwt. 16 gr.

13. From 3 mi. 1400 yd. 2 ft. 6 in. subtract 1 mi. 1500 yd. 1 ft. 9 in.

14. How long will 9 bu. 1 pk. 4 qt. of oats last a horse if he is fed 3 times each day and receives 4 qt. each time?

15. Bought $1\frac{1}{2}$ bu. of chestnuts at \$ 2 a bushel and sold them at 5¢ a pint. How much did I gain?

16. How many spoons, each weighing 2 oz. 5 pwt., can be made from 6 lb. 4 oz. 10 pwt. of silver?

17. Find the value of 3 lb. 4 oz. of gold at \$ 20 an ounce.

18. At \$12 a ton, what is the cost of 6 T. 1500 lb. of hay?

19. At 18¢ a pound, what is the cost of 2 lb. 6 oz. of cheese?

20. Paid 15¢ for half a peck of potatoes. What was the rate of this price per bushel?

21. How long will it take to travel 54 miles, at the rate of 4.5 miles per hour?

22. A barrel of oil containing 50 gallons was sold for \$ 5.75. What was the price per quart?

23. How many tons of steel rails will it take for a single-track railroad per mile, if the rails average 70 lb. to the yard?

24. How many tons will 500 bushels of potatoes weigh?

25. How many bushels in 6 tons of oats?

26. When vinegar is 6¢ a quart, how much must be paid for $\frac{7}{8}$ gal.?

27. Find the cost of 8 yd. 1 ft. 6 in. of pipe, 4 lb. to the foot, at 25¢ a pound.

28. Bought lead pencils at $3.50 a gross and sold them at 5¢ each. How much did I make on each gross?

29. How many bushels in 10 tons of wheat?

30. Find the time from June 6, 1896, to Jan. 5, 1910.

31. What decimal of a mile is 80 rods?

32. At $6.50 a ton, what is the cost of 5600 lb. of coal?

33. Bought paper at $3.00 per 1000 sheets and sold it at 15¢ a quire. How much did I make on each 1000 sheets?

34. A yard is $\frac{3600}{3937}$ of a meter. How many yards are 500 meters?

35. How many meters in a mile?

36. Bought potatoes by the car load at $16 a ton and sold them at 60¢ a bushel. Did I make or lose, and how much on 10 tons?

37. Bought nails at $3.75 per keg and sold them at $4\frac{1}{2}$¢ a pound. How much did I make or lose on each keg?

38. A train running at the rate of 50 miles per hour passes over how much ground each second?

39. What part of the circumference of a circle is an arc of 60°?

40. A note is given on Feb. 12, 1909, for 2 yr. and 6 mo. At what date will this note be due?

41. What part of a mile is 10 rods?

42. Change 3 pk. 4 qt. to the fraction of a bushel.

43. How many steps, each 2 ft. 6 in. long, will a boy take in walking $\frac{1}{2}$ mile?

44. What is the weight of 600 bushels of potatoes in tons?

45. At \$.89 a yard, how many yards of cloth can be bought for \$10?

46. A bin that will hold 80 bushels of oats, stricken measure, will hold how many bushels of potatoes, heaped measure.

47. A bin that will hold 80 bushels of apples will hold how many bushels of wheat?

48. What is the value of 15 pwt. of gold dust at \$20 per ounce?

49. A man measured the angle at the corner of his garden and found it to be 95° 15′. Was this more or less than a right angle, and how much?

50. From 2 miles 75 rods subtract 1 mile 150 yards.

MEASUREMENT OF PLANE SURFACES

121. A plane surface is a flat surface, like the floor of the room. It has extent but no thickness.

122. $\angle$ angle. An angle is the opening formed by the separation of two lines meeting at a point.

123.

right angle	right angle
right angle	right angle

A right angle includes $\frac{1}{4}$ of the space about a point on a flat surface. A square corner is a right angle.

124. Angular space is measured in degrees, the entire space about a point on a plane surface containing 360 degrees. A right angle, therefore, contains 90 degrees.

125. A plane figure bounded by four straight lines is called a quadrilateral. (Quadrilateral means four-sided.)

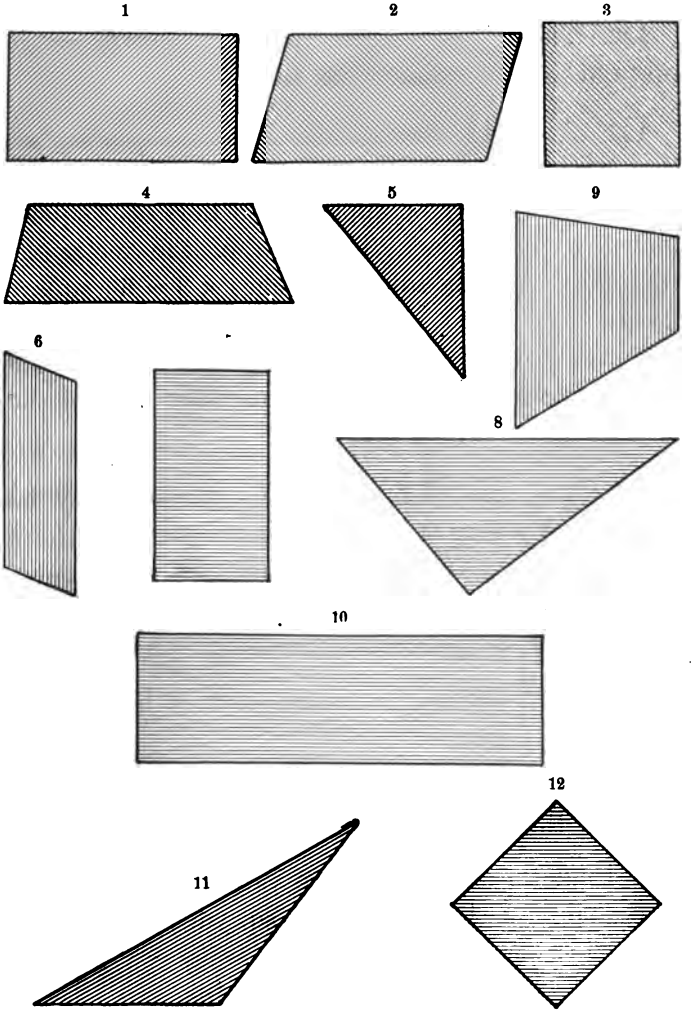
126. A parallelogram is a quadrilateral whose opposite sides are parallel.

127. A rectangle is a parallelogram having right angles.

128. A square is a rectangle having equal sides.

129. A triangle is a plane surface having three straight sides.

Copy the figures shown below, apply the definitions given above, and then name each of them.



131. A trapezoid is a quadrilateral two of whose sides are parallel.

132. The area of a surface is the number of square units of measure that it contains.

133. A rectangular surface 2 inches by 3 inches has the square inch for its unit of area measure. This measure may be applied in 3 tiers of 2 square inches to each tier. The area of the rectangle, therefore, is 2 square inches $\times$ 3 = 6 square inches.

(Make a drawing to show the application of the square unit of measure in the measurement of the area of the rectangle given.)

134. The area of a rectangular surface may be found by applying the square unit of measure to its surface until such surface is measured. In computing the areas of rectangular surfaces, we express in square units the product of the length and breadth.

1. Find the area of a rectangle 4 in. long and 3 in. wide.

4 sq. in. $\times$ 3 = 12 sq. in. *Ans.*

(Show the full measurement by diagram.)

2. Find the area in square yards of a rectangle 18 ft. long and 4 ft. wide. (1 sq. yd. = 9 sq. ft.)

18 sq. ft. $\times$ 4 = 72 sq. ft.

72 sq. ft. = 8 sq. yd. *Ans.*

3. What is the area of the floor of a room which is 24 ft. long and 15 ft. wide?

4. What is the area of a sidewalk which is 80 ft. long and $5\frac{1}{2}$ ft. wide?

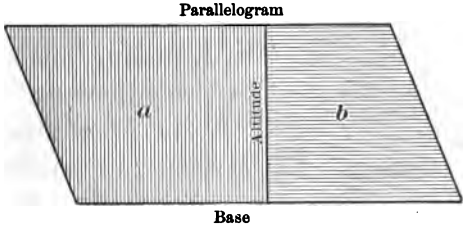
5. What is the area of a square which is 24 ft. on a side?

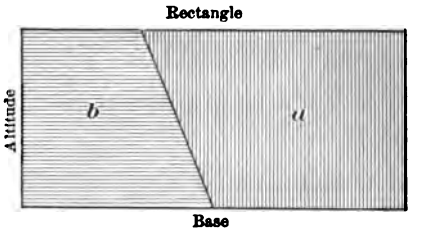
135. The base of a plane figure or flat surface is the side upon which it is assumed to stand. Any side of the figure may be considered its base.

136. The altitude of a plane figure is its height measured on a line perpendicular to its base.

137. The area of a parallelogram is equal to that of a rectangle having the same base and altitude. The area of a parallelogram is measured, therefore, by the product of its base and altitude, the product being expressed in square units.

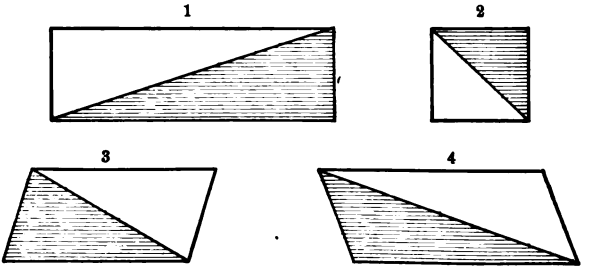
To prove that a parallelogram is equivalent to a rectangle having the same base and altitude, cut out a parallelogram from a piece of paper, draw its altitude, and then cut along the line which marks the altitude and apply the pieces as shown in the diagram.



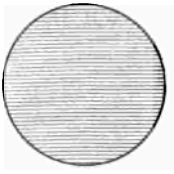


Prove this by cutting paper and comparing the triangular pieces.

138. The area of a triangle is $\frac{1}{2}$ that of a rectangle or parallelogram having the same base and altitude.



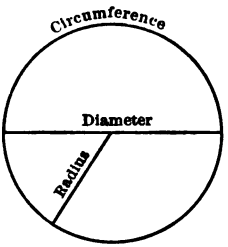
139. A circle is a plane figure bounded by a curved line, every point of which is equally distant from the center of the circle.



140. The bounding line of a circle is called its circumference.

The diameter of a circle is a straight line passing through its center and ending in its circumference.

The radius of a circle is a straight line drawn from its center to its circumference.

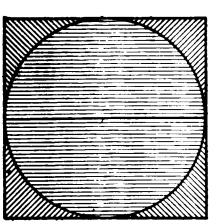


141. The circumference of a circle is 3.1416 times its diameter. $3\frac{1}{7}$ may be used for 3.1416 for all practical purposes.

142. The area of a circle is $\frac{1}{4}$ of the product of its diameter and its circumference, expressed in square units.

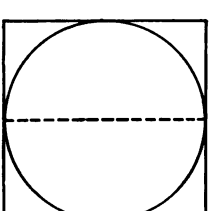
The circle may be considered as composed of an infinite number of triangles, the sum of whose bases equals the circumference, and whose altitudes are the radius of the circle. The rule for finding the area of the circle given above is based upon this conception.

143. The area of a circle is less than the area of a square having the same diameter.



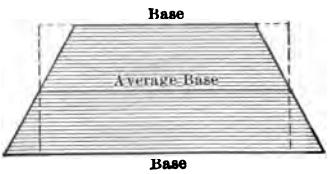
The area of the circle = .7854 of the area of the square having the same diameter.

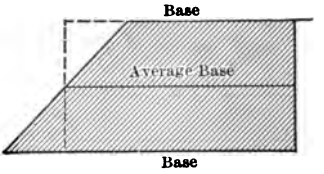
144. The circumference of the circle is less than the perimeter of the square having the same diameter.

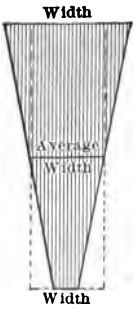


The circumference of the circle is .7854 of the perimeter of the square having the same diameter.

145. Circumference = diameter × 3.1416.
Diameter = circumference ÷ 3.1416.
Area of circle = diameter × $\frac{1}{4}$ of circumference.
Area of circle = diameter × diameter × .7854.







146. The area of a trapezoid is found by multiplying the average length of the two parallel sides by the perpendicular distance between them, and expressing the result in square units.

The average length of the two sides is $\frac{1}{2}$ of their sum.

147. The dimensions of surfaces are given in linear units, which have length only.

The area of surfaces is measured in square units which have length and breadth.

LINEAR UNIT

SQUARE UNIT

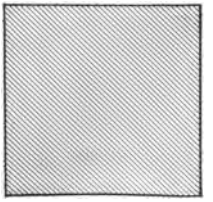


TABLE FOR MEASUREMENT OF AREAS

148. 144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)

UNIT EQUIVALENTS

Acre		Sq. Rd.		Sq. Yd.		Sq. Ft.		Sq. In.
						1	=	144
				1	=	9	=	1296
		1	=	$30\frac{1}{4}$	=	$272\frac{1}{4}$	=	39204
1	=	160	=	4840	=	43560	=	6272640

Square rods and acres are only used in measuring large land areas. Small areas of land are measured in square yards or square feet.

Exercises and Applications in Surface Measure

149. 1. Find the area in square feet of a rectangular lot 180 ft. by 75 ft.

180 sq. ft. × 75 = —— sq. ft. *Ans.*

2. Find the area of a parallelogram whose base is 64 yd. and altitude 50 yd.

64 sq. yd. × 50 = —— sq. yd. *Ans.*

3. Find the area of a field in the form of a trapezoid whose parallel sides are 60 yd. and 80 yd. and the perpendicular distance between them 45 yd.

Ans. —— sq. yd.

$\frac{60 + 80}{2} \times 45 =$ ——, which is the number of square yards in its area.

4. Find the area of a triangle whose base is 18 ft. and its altitude 12 ft. *Ans.* —— sq. ft.

$$\frac{18 \text{ sq. ft.} \times 12}{2} = \text{——} \text{ sq. ft.}$$

5. How many acres in a field 80 rd. by 60 rd.?

80 sq. rd. × 60 = 4800 sq. rd.

4800 sq. rd. = (4800 ÷ 160) acres.

6. Reduce 30 A. 5 sq. rd. to square rods.

7. Reduce to higher denominations 7640 sq. in.

8. What part of a square yard is 648 sq. in.?

9. What part of an acre is 60 square rods?

10. How many rods of fence will be needed to inclose a field 60 rods long and 40 rods wide? What is the area of the field in acres?

11. What is the depth of a lot that has 120 feet front and contains 18,720 square feet?

12. Find the cost of a farm 198 rd. long and 150 rd. wide at $80 an acre.

13. How many square yards in a triangle whose base is 148 feet and its altitude 45 feet?

14. What length of tire will it take to band a wagon wheel which is 5 feet in diameter?

15. At $85 an acre, what is a rectangular farm worth which is 75 rd. wide and 80 rd. long?

16. A lot is 100 ft. deep and 40 ft. front. How many square yards does it contain? What will it cost to fence it at 50¢ a foot?

17. A 20-acre field is 80 rods long. How wide is it?

18. The altitude of a triangular field is 32 yards and its area 640 square yards. What is the length of its base?

19. Find the area in square feet of a flower bed 20 ft. 8 in. in length and 10 ft. 6 in. in width.

20. Find the cost of painting both sides of a tight board fence 50 ft. long and 6 ft. high at 15¢ a square yard.

21. How much will it cost to cement the floor of a cellar 40 ft. by 20 ft. at 90¢ a square yard?

22. If it requires 72 rods of fence to inclose a square field, how long is a side of the field? How many acres does the field contain?

23. What is the circumference of a circle whose diameter is 14 ft.? What is its area in square feet?

24. How many square feet in the surface of a blackboard which is 2 ft. 6 in. wide and 18 ft. long?

25. A boy wishing to know the diameter of a flag pole measured around it with a string, then measured the string and found it to be $25\frac{1}{8}$ inches. What was the diameter of the pole?

26. What is the area in square feet of a square flower bed 4 ft. on a side? How much less is the area of a circular bed which is 4 ft. in diameter?

27. What is the area in square yards of a tennis court which is 36 ft. by 78 ft.?

28. What would it cost to turf such a court at 10¢ a square yard?

29. What will a cement walk cost that is 80 ft. long and $5\frac{1}{2}$ ft. wide at 18¢ a square foot?

30. How many rods of fence will it take to inclose a field which is 80 rods long and 30 rods wide?

31. How many feet of picture molding will it take to go entirely around a room which is 15 ft. by 12 ft.?

32. How long a piece of hoop iron will it take to go around a circular tank which is 6 feet in diameter?

33. How far must a wagon move to turn a wheel once around that is 5 feet across?

34. How many times would such a wheel turn if the wagon traveled $\frac{1}{2}$ mile?

35. A piece of board is 12 inches wide at one end and 10 inches wide at the other and 14 feet long. How many square feet in one of its surfaces?

36. A village lot is 100 feet deep, the front and rear lines being parallel. The front is 80 feet and the rear 70 feet. How many square feet does the lot contain?

37. The driving wheel of a locomotive is 7 feet in diameter. How many times will such a wheel turn while the locomotive is running 10 miles?

38. What will it cost to cement the sides and bottom of a circular cistern which is 5 feet in diameter and 6 feet in depth if it costs 10¢ a square foot for the cement?

39. Which is the larger, a square field with sides of 40 yd. or a rectangular field with sides of 80 yd. and 20 yd.?

40. How many yards of fence will it take to inclose each of the fields named in the last example?

41. If telegraph poles are set 10 rods apart, how many will it take for 25 miles?

42. If fence posts are set 8 feet from center to center, how many will be needed for a front fence 80 feet long? (Make a sketch so as to get the number correct.)

43. How many times larger is a circle 4 feet in diameter than one which is 2 feet in diameter?

44. In some sections sidewalks are made of bricks, the bricks being 8 in. long, 4 in. wide, and 2 in. thick. How many such bricks will it take for a walk 4 ft. wide and 40 ft. long, if the bricks are laid on their sides? How many bricks will it take if the bricks are laid on their edges?

45. A boy drives a peg into the ground and fastens a string to it, that he may lay out a circular flower bed. What is the area of the bed if he use a string 5 ft. long?

46. After traveling 1650 yards, how many feet remain to travel to cover a distance of .1 mile?

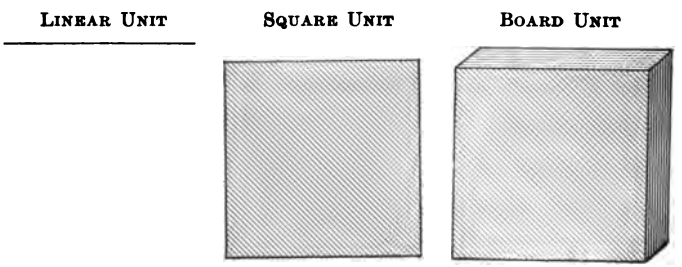
47. How many square yards of paving will it take to pave a street which is 50 feet wide and 250 feet long?

48. What part of an acre is contained in a lot which is 5 rods wide and 7 rods long?

49. Find the diameter of a circle whose circumference is 500 feet.

BOARD MEASURE

150. Board measure is used in measuring lumber. The unit of measure is the board foot, which is 1 foot square and 1 inch or less in thickness.



Boards less than one inch thick are measured as if they were one inch thick. Planks or timbers more than one inch thick are measured as if they were composed of one-inch boards.

A board 14 feet long, 1 foot wide, and 1 inch thick contains 14 board feet of lumber. If only $\frac{3}{4}$ or $\frac{7}{8}$ inch thick, it would still be measured as

14 board feet. If it were $1\frac{1}{2}$ inches thick, it would contain $1\frac{1}{2}$ times 14 board feet; and if 2 inches thick, 2 times 14 board feet, etc.

Applications of Board Measure

151. 1. How many board feet of lumber in a board 18 ft. long, 16 in. wide, and 1 in. thick?

$\frac{16}{12}$ bd. ft. $\times 18 = 24$ bd. ft. *Ans.*

2. How many board feet of lumber in a stick of timber 20 ft. long, 10 in. wide, and 8 in. thick?

$\frac{10}{12}$ bd. ft. $\times 20 \times 8 =$ —— bd. ft. *Ans.*

3. Find the number of board feet of lumber in 4 planks, each 16 ft. long, 10 in. wide, and 3 in. thick.

4. Find the number of board feet of lumber in 12 scantlings, each 16 ft. long and 3 by 4 in.

5. What will it cost, at \$ 30 per 1000 feet for the lumber, to lay a barn floor 18 ft. wide, 30 ft. long, with planks 2 in. thick?

6. How many board feet of lumber will it take to floor a bridge that is 60 ft. long, if the floor is laid 12 ft. wide and made of 3-in. plank?

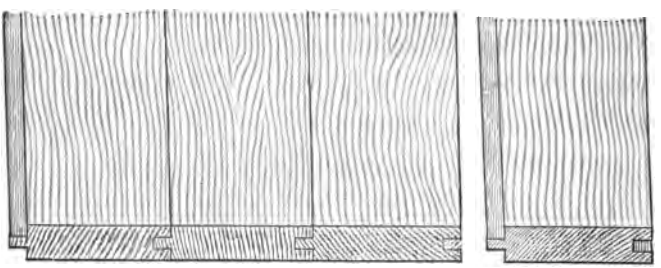
7. Find the cost at \$ 35 per M of 50 joists, each 2 in. by 8 in. and 20 ft. long.

Making Allowance for Matching in Flooring, Wainscoting, Ceiling, Siding for Barns, etc.

152. Floors of rooms, wainscoting, ceilings, and barn siding are often laid with "tongue-and-

grooved" or matched lumber, as shown in the diagram below. This takes more lumber, as the "tongue" has to be included in the measurement of the width of the board.

For narrow boards this proportion is greater than for wider boards, as the "tongue" remains the same for all widths.



1. How many board feet of 1-inch lumber will it take to lay a kitchen floor 12 by 14 ft. if $\frac{1}{8}$ is allowed for matching?

12 bd. ft. $\times$ 14 $\times \frac{9}{8} =$ —— bd. ft. *Ans.*

2. How many board feet of lumber will it take to floor a platform 18 ft. wide and 20 ft. long, the flooring being 1 in. thick, allowance of $\frac{1}{7}$ being made for matching and waste?

3. How many board feet of wainscoting will it take to go around a kitchen which is 14 ft. long and $12\frac{1}{2}$ ft. wide, if the wainscot be made 3 ft. high from the floor and laid with $\frac{7}{8}$-in. lumber and allowance of $\frac{1}{8}$ for matching be made?

4. How many board feet of $\frac{7}{8}$-in. lumber will it take to make 2 doors for a barn, each door to be 9 ft. long and 7 ft. wide, and to be made of matched pine boards $2\frac{1}{2}$ in. wide? (Allow $\frac{1}{5}$ for matching.)

5. How many board feet of $\frac{3}{4}$-in. lumber will it take for the ceiling of a room which is 14 ft. wide and 16 ft. long, if the boards are 2 in. wide and "tongue-and-grooved"? (Allow $\frac{1}{4}$ for matching.)

6. How many board feet of 1-in. lumber will it take for the siding of a barn which is 30 ft. wide and 40 ft. long and 18 ft. high on sides, if the boards are matched and laid 4 in. wide, the gable of the barn rising 10 ft. on the ends? (See dictionary for "gable," and make allowance of $\frac{1}{8}$ for matching.)

7. How many board feet of lumber will it take to make a tight board fence of matched pine board 1 in. thick and laid 6 in. wide, if the fence is 50 ft. long and 6 ft. high? (Allow $\frac{1}{12}$ for matching.)

8. How many board feet of 1-in. matched lumber will it take to cover the sides of a silo which is cylindrical in form, 10 ft. in diameter, and 16 ft. in height? (Allow $\frac{1}{8}$ for matching.)

Note. For straightening and matching a board about $\frac{1}{2}$ inch is used, and this gives a base for comparison for the allowance to be made. Loss of $\frac{1}{2}$ inch in each 4 inches means an allowance of $\frac{1}{8}$ for matching, etc.

MEASURE OF VOLUME

153. Volume or solid contents has three dimensions: length, breadth, and thickness. The unit of measure for volumes is a cube.

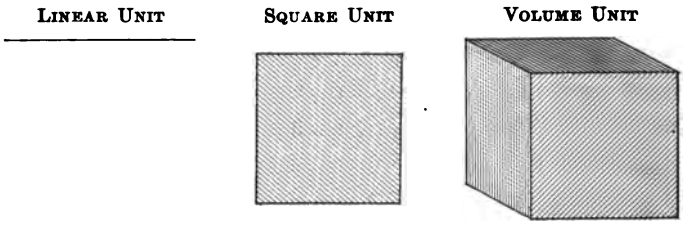


TABLE FOR CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Note. In measuring wood, 128 cu. ft. is sometimes called a cord, but more often a pile 8 ft. long and 4 ft. high is called a cord, and the length of the wood named. A cord of 4-ft. wood contains 128 cu. ft., but a cord of 2-ft. wood is often estimated as described and contains, if thus estimated, only 64 cu. ft.

A cord of 18-in. wood contains, if estimated by the pile of 8 ft. in length and 4 ft. in height, only 48 cu. ft. Custom varies in different sections of the country about the measurement of wood.

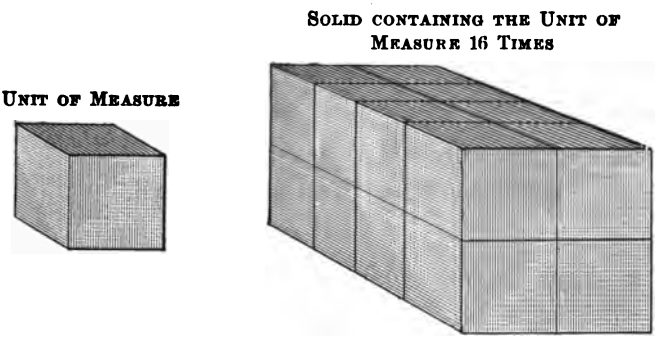
154. In measuring the contents of bins in bushels, 2150.42 cubic inches for a bushel are used.

In estimating the contents of bins in bushels, 1.25 cubic feet to the bushel is sufficiently accurate.

155. In measuring the contents of tanks in gallons, 231 cubic inches to the gallon are used.

In estimates 7.5 gal. to the cubic foot is sufficiently accurate.

156. Finding the Cubic Contents or Volume of Rectangular Solids



157. The volume of a rectangular solid is found by applying the unit of measure to each of its three dimensions and finding the product of the results.

In the figure shown above the unit of measure may be applied to the width 2 times, to the height 2 times, and to the length 4 times. The solid, therefore, contains the unit of measure $2 \times 2 \times 4$ or 16 times.

If the dimensions of the solid were given in inches, the unit of measure would be a cubic inch, and the volume of the solid 16 cubic inches. If the dimensions were given in feet or yards, the unit of measure and the volume would be named in corresponding terms of cubic measure.

Applications of Cubic Measures

158. 1. How many cubic feet in a rectangular stone wall, which is 2 ft. thick, 5 ft. high, and 16 ft. long?

2 cu. ft. × 5 × 16 = —— cu. ft. *Ans.*

2. How many cubic yards of earth will be removed in digging a cellar 24 ft. wide, 40 ft. long, and 6 ft. deep?

24 cu. ft. × 40 × 6 = 5760 cu. ft.

5760 cu. ft. = (5760 ÷ 27) cu. yd.

= 213⅓ cu. yd.

3. How many cubic yards of earth will be needed to raise the surface of a lawn 6 in., if the lawn is 30 ft. wide and 36 ft. long?

½ cu. ft. × 30 × 36 = 540 cu. ft.

540 cu. ft. = (540 ÷ 27) cu. yd. = 20 cu. yd.

4. How many cubic feet of water will a tank hold which is 18 in. deep, 4 ft. wide, and 10 ft. long?

5. How many stricken bushels of grain will a bin hold whose inside dimensions are 4½ ft. by 3 ft. by 8 ft.?

6. How many bushels of apples will such a bin hold?

7. What is the volume in cubic yards of a rectangular block of stone 18 in. by 24 in. and 9 ft. long?

8. My sleeping room is 10 ft. long, 9 ft. high, and 8 ft. wide. How many cubic feet of air does it contain?

9. How many gallons of water will a tank contain that is 4 ft. long, $3\frac{1}{2}$ ft. wide, and 18 in. deep?

10. A coal bin is 22 ft. long, 12 ft. wide, and 6 ft. deep. How many tons of coal will it hold, allowing 35 cu. ft. to the ton?

11. In a tank which is 4 ft. wide and 6 ft. long the water is 9 in. deep. How many gallons of water in the tank?

12. A wagon box is 2 ft. 9 in. wide and 10 ft. long. How high must the sides be to hold 1 cu. yd. of earth?

13. A stone wall is 2 ft. thick at the bottom and 1 ft. thick at the top. It is 5 ft. high and 100 ft. long. How many cubic yards of stone does it contain? (Use the average thickness in the computation.)

14. How many bricks, allowing 22 to the cubic foot, will be required to build a brick wall 16 in. thick, 4 ft. high, and 18 ft. long?

15. How high will a bin which is 7 ft. wide and 8 ft. long have to be to hold 10 T. of coal? (Estimate volume of each ton of coal as 35 cu. ft.)

16. How much water will a cistern hold which is 4 ft. square and 5 ft. deep?

17. How much water will a cistern hold which is cylindrical and 4 ft. in diameter and 5 ft. deep?

(The volume of a cylinder is .7854 of the volume of a square prism of like dimensions.)

$4 \times 4 \times .7854 \times 5 =$ ——, the number of cubic feet.

—— cu. ft. = —— gal.

18. How much water will a cylindrical boiler, such as is used in kitchens, hold if it is 12 in. in diameter and 5 ft. in length?

19. How many gallons of water will a circular tank hold which is 10 ft. in diameter and has perpendicular sides 2 ft. in height?

20. A silo is often made cylindrical in shape. How many cubic feet of space in such a silo which is 20 ft. in diameter and 15 ft. in height?

21. The water in a well which is cylindrical in shape and 3 ft. in diameter is 10 ft. deep. How many barrels of water in the well?

22. How much earth must be removed in digging a well 5 ft. in diameter and 20 ft. in depth, if it is cylindrical in shape?

23. Cast iron weighs about 450 lb. to the cubic foot. What is the weight of a rectangular piece of cast iron 3 in. thick, 4 ft. wide, and 6 ft. long?

24. What is the weight of a cylindrical bar of steel 2 in. in diameter and 6 ft. long? (Steel weighs about 490 lb. to the cubic foot.)

25. What is the weight of a rectangular piece of wrought iron 2 inches square and 6 feet long? (Wrought iron weighs about 480 pounds to the cubic foot.)

SUMMARY OF DENOMINATE NUMBERS

1. A denominate number is a concrete number in which the unit of measure has a fixed value established by law or custom.

2. The operations of reduction, addition, subtraction, multiplication, and division of denominate numbers are like the corresponding operations with simple numbers, except that the scale of increasing and decreasing units varies in different measures in denominate numbers and is uniformly 10 in simple numbers.

3. Liquid measure is used in measuring liquids. The standard unit is the gallon of 231 cubic inches.

4. Dry measure is used in measuring fruit, grain, vegetables, etc. The standard unit is the bushel of 2150.42 cubic inches. Coarse vegetables, etc., are measured by the heaped bushel, which contains $\frac{5}{4}$ of the standard bushel.

5. Linear measure is used in measuring lengths and distances. The standard unit is the yard of $\frac{3600}{3937}$ meters.

6. Avoirdupois weight is used for all ordinary purposes of weighing. The standard unit is the pound of 7000 grains.

7. Troy weight is used in weighing gold and silver. The standard unit is the pound of 5760 grains.

8. The weight of diamonds and pearls is often expressed in carats. The carat thus used is equal to 3.168 grains, Troy.

9. The term "carat" or "karat" is also used to express the proportion of gold to the total weight in gold rings, etc. 24 is used as the standard. 18 karats gold means $\frac{18}{24}$ gold.

10. Angular space and arcs of circles are measured by circular measure. The unit is a degree.

11. A plane surface or a plane is a flat surface like the floor of a room.

12. An angle is the difference in direction of two lines meeting at a point.

13. A right angle contains 90 degrees. It is often called a square corner.

14. A quadrilateral is a four-sided figure.

15. A parallelogram is a quadrilateral whose opposite sides are parallel.

16. A rectangle is a parallelogram having right angles.

17. A square is a rectangle having equal sides.

18. A triangle is a plane surface having three straight sides.

19. A trapezoid is a quadrilateral two of whose sides are parallel.

20. The area of a plane surface is the number of square units of measure that it contains.

21. The area of a rectangular surface is equal to the product of its length and width.

22. The base of a figure is the side upon which it is assumed to stand. Any side may be considered as a base.

23. The altitude of a figure is its height measured on a line perpendicular to its base.

24. The area of a parallelogram is equal to the product of its base and altitude.

25. The area of a triangle is equal to $\frac{1}{2}$ the product of its base and altitude.

26. A circle is a plane surface bounded by a curved line every point of which is equally distant from the center of the circle.

27. The circumference of a circle is its bounding line.

28. The diameter of a circle is the distance across it, measured through the center.

29. The radius of a circle is the distance from its center to its circumference.

30. The circumference of a circle is 3.1416 times its diameter.

31. The area of a circle is equal to $\frac{1}{4}$ the product of its diameter and its circumference. It is also equal to .7854 of the square of its diameter.

32. The area of a trapezoid is equal to the aver-

age of its parallel sides multiplied by the perpendicular distance between them.

33. The dimensions of surfaces is given in linear units, but the area can only be expressed in square units.

34. The area of a circle is .7854 that of a square of like diameter.

35. The volume of a cylinder is .7854 that of a square prism of like dimensions.

36. Board measure is used in measuring lumber. The unit is a board foot, which is 1 foot square and 1 inch thick.

37. In the measure of volume there are three dimensions, length, breadth, and thickness. The unit of measure has these three dimensions equal and is a cube. This measure is also called cubic measure.

38. In measuring the capacity or contents of bins in bushels, 2150.42 cu. in. is used for a bushel.

39. In measuring the capacity or contents of tanks, etc., in gallons 231 cu. in. is used for a gallon.

40. In estimating the capacity of bins for coal of domestic sizes, 35 cu. ft. to the ton is used.

41. The cubic capacity, contents, or volume of a rectangular body is equal to the product of its three dimensions, but can only be expressed in terms of cubic measure.

CHAPTER IX

PERCENTAGE

159. A percentage of a number is the result obtained by taking a stated number of hundredths of it.

160. The term "per cent" means by the hundred.

161. The sign % is used by business men for the words "per cent."

The rate per cent expressed in exercises or problems must be changed to decimal or fractional form before it can be used in computation.

To find 25% of \$60, we use the rate as .25, or as $\frac{1}{4}$ in the computation, as follows:

\$60 × .25 = \$15; or \$60 × $\frac{1}{4}$ = \$15.

EQUIVALENT PER CENT, DECIMAL AND FRACTIONAL EXPRESSIONS

162.

Per cents	Decimals	Fractions	Per cents	Decimals	Fractions
100%	= 1.00	= 1	$83\frac{1}{3}$%	= $.83\frac{1}{3}$	= $\frac{5}{6}$
50%	= .50	= $\frac{1}{2}$	$12\frac{1}{2}$%	= .125	= $\frac{1}{8}$
$33\frac{1}{3}$%	= $.33\frac{1}{3}$	= $\frac{1}{3}$	$37\frac{1}{2}$%	= .375	= $\frac{3}{8}$
$66\frac{2}{3}$%	= $.66\frac{2}{3}$	= $\frac{2}{3}$	$62\frac{1}{2}$%	= .625	= $\frac{5}{8}$
25%	= .25	= $\frac{1}{4}$	$87\frac{1}{2}$%	= .875	= $\frac{7}{8}$
75%	= .75	= $\frac{3}{4}$	$\frac{1}{2}$%	= .005	= $\frac{1}{200}$
20%	= .20	= $\frac{1}{5}$	$\frac{1}{4}$%	= .0025	= $\frac{1}{400}$
60%	= .60	= $\frac{3}{5}$	$\frac{1}{8}$%	= .00125	= $\frac{1}{800}$
$16\frac{2}{3}$%	= $.16\frac{2}{3}$	= $\frac{1}{6}$			

The table given on page 139 shows all of the per cent expressions in common use. The decimal or fractional value of any per cent expression may be easily found, if we note that the sign may be removed, and the decimal point placed two places further to the left, in changing any per cent expression to decimal form.

$.3\% = .003$; $3\% = .03$; $30\% = .3$; $300\% = 3$.

163. The general applications of percentage may be conveniently grouped into three classes, as follows:

1. Find 8% of $400.

$$\$400 \times .08 = \$32. \quad \textit{Ans.}$$

2. If 8% is $32, what is 100% or the entire quantity?

$$\$32 \div .08 = \$400. \quad \textit{Ans.}$$

3. $32 is what per cent of $400?

$$\$32 \div 400 = .08. \quad .08 = 8\%. \quad \textit{Ans.}$$

164. The number on which the percentage is computed is called the **base**. The careful determination of the number which represents the base is the **key** to all applications of percentage.

Per cent expressions, in themselves, have no definite value, as they only express a certain rate. The base must be known before the value of the per cent expression becomes definite.

"A man lost 25% of his money," means that he

lost $\frac{1}{4}$ of it; but we cannot tell how much money he lost, until we know the base, which was the sum he had at first. "A man having \$ 80, lost 25 % of it. How much money did he lose?" becomes definite, as we know the base.

CLASS ONE IN GENERAL APPLICATIONS OF PERCENTAGE

Finding a Percentage of a Number

165. Find:

1. 3 % of 400.
2. 5 % of \$ 150.
3. 60 % of 300 bushels.
4. 8 % of 75.
5. 25 % of 800.
6. 150 % of 3500.
7. $12\frac{1}{2}$ % of 640.
8. 75 % of 160 acres.
9. $87\frac{1}{2}$ % of 84.
10. $62\frac{1}{2}$ % of \$ 960.
11. 30 % of \$ 540.
12. 40 % of 750 bushels.
13. 60 % of \$ 1200.
14. 80 % of \$ 90.
15. $1\frac{1}{2}$ % of \$ 480.
16. $\frac{1}{2}$ % of \$ 600.
17. $\frac{1}{8}$ % of \$ 720.
18. $4\frac{1}{8}$ % of \$ 2400.
19. $3\frac{1}{4}$ % of \$ 360.
20. $\frac{3}{8}$ % of \$ 7200.
21. $\frac{7}{8}$ % of \$ 160.
22. 250 % of \$ 50.

23. A man had 150 sheep and sold 40 % of them. How many sheep did he sell?

24. In a spelling lesson of 40 words, a boy spelled 80 % correctly. How many words did he spell correctly?

25. A bin which will hold 90 bushels of grain is 75 % filled. How many bushels of grain in the bin?

26. A piano cost $ 675. If, after using it for a short time, I sell it at a loss of 25 % of its cost, how much do I receive for it?

27. If wheat produces 80 % of its weight in flour, how many pounds of flour can be made from 10 bushels of wheat?

28. A cubic foot of water weighs 62½ lb. The weight of ice is about 92 % of the weight of water. Find the weight of a cubic foot of ice.

29. Find the weight of a block of ice 18 in. square and 50 % of its width thick.

30. A man's income is $ 4500 a year and he saves 12½ % of it. How much does he save each year?

31. The width of a floor is 75 % of its length. If its length is 24 feet, how many square feet of surface has the floor?

32. A train running 40 miles an hour increases its speed 20 %. How long will it take it to run 90 miles under the increased rate of speed?

33. A man rents a house at $ 25 a month. The second year the rent is advanced 20 %. What is his year rent after the advance?

34. How many board feet of inch lumber will it take to lay a floor 24 ft. by 18 ft., if an extra

allowance of 25% is made for matching and waste?

35. By purchasing his feed by the car load, a farmer saves 20% of the cost. If he paid $250 a year for feed when bought in small lots, how much does he save by purchasing by the car load?

CLASS TWO IN GENERAL APPLICATIONS OF PERCENTAGE

Finding the Number when the Percentage and the Rate Per Cent are Given

166. 1. A man lost $72, which was 6% of all his money. How much money did he have at first?

6% of his money = $72.
1% of his money = $12.
100% of his money = $1200. *Ans.*

SHORT SOLUTION

$72 ÷ .06 = $1200.

2. An agent charges 3% for selling my house, charging $45. What was the selling price of the house?

3% of selling price = $45.
Selling price = $45 ÷ .03 = $1500. *Ans.*

3. A man gives $125 for a church debt. This was 10% of his yearly income. What was his income?

4. $900 is 15% of what number?

5. A man pays $80 on a debt, which sum was 20% of the debt. How much remained to be paid?

6. 25% of my income is $1000. What is my income?

7. The daily attendance in a school is 2400, which is 96% of the registered number of pupils. How many pupils are registered?

8. I sold my house for $4578, which was 84% of its cost. What was its cost?

9. My salary is $250 a month, which is 25% more than it was last year. What was my salary per month last year?

100% of last year's salary = last year's salary.
125% of last year's salary = present salary or $250.

$250 ÷ 1.25 = $200. *Ans.*

10. I pay $8 a week for my board, which is 20% less than I paid last year. How much per week did I pay last year?

100% of last year's board = last year's board.
80% of last year's board = present board or $8.

$8 ÷ .80 = $70. *Ans.*

11. After spending 80% of his money, a man had $400 left. How much money had he at first?

100% of his money = his money.
100% − 80% = 20%.
20% of his money = $400.
100% of his money = $400 ÷ .20 = $2000. *Ans.*

12. A farmer sold 50 sheep, which was 5 % of his whole flock. How many sheep had he left?

13. A man sold 8 % of his sheep and had 368 left. How many had he at first?

14. The weight of marble is about $33\frac{1}{3}$ % of the weight of wrought iron. If marble weighs 160 lb. to the cubic foot, what is the weight of 3 cu. ft. of wrought iron?

15. A cubic foot of ice weighs about 8 % less than a cubic foot of water. If a cubic foot of ice weighs 57.5 lb., what is the weight of a cubic foot of water?

16. Lead is about 44.9 % heavier than steel. If a cubic foot of lead weighs 710 lb., how much does a cubic foot of steel weigh?

17. The growth in population in ten years in a city was 12 %. Its present population is 224,000. What was its population 10 years ago?

18. What number increased by 18 % of itself is equal to 1475?

19. What number diminished by 18 % of itself is equal to 6150?

20. 20 % of a farm is woodland. If there are 75 acres of woodland, how many acres are there in the farm?

21. My commission at 3 % for selling real estate was $36. How much real estate did I sell?

22. My commission as a broker for selling wheat at $\frac{1}{8}\%$ commission was $15. What was the value of the wheat sold?

23. A reduction at 20% on the price of a house reduced the cost $1500. What was the original price of the house?

24. An advance of 5% on the price of butter raised the selling price to 21 cents a pound. What was the price per pound before the advance?

25. My taxes on a house at 1.4% were $70. Upon what amount was this tax paid?

26. A storm damaged a field of wheat 10% of its entire value. If the loss was $75, what was the original value of the crop of wheat?

27. A farm has increased in value 5%, and is now worth $2992.50. What was the value of the farm before the increase?

28. By the drop in the market value of some stocks a man lost $720. If the drop was $\frac{1}{4}\%$ of the cost of the stock, what was its cost?

29. The daily attendance in a certain school was 6% more in October than in September. If it was 2597 in October, what was the average daily attendance in September?

30. At a forced sale, a horse was sold at 10% less than its cost, selling for $117. What was the cost of the horse?

CLASS THREE IN GENERAL APPLICATIONS OF PERCENTAGE

Finding what Per Cent One Number is of Another

(See Arts. 96 and 97 of Fractions.)

167. 1. What part of 35 is 7? What per cent of 35 is 7?

2. What part of 480 is 24? What per cent of 480 is 24?

3. What per cent of 300 is 75?

4. What per cent of 1472 is 460?

$460 \div 1472 = .3125 = 31.25\ \%$. *Ans.*

5. A man having 375 bushels of potatoes, sold $37\frac{1}{2}$ bushels. What per cent did he sell?

6. In a spelling lesson of 50 words, a boy spelled 45 words correctly. What per cent of the words did he miss?

7. A baseball team played 15 games and won 9 of them. What per cent of its games did it win?

8. Water weighs 62.5 lb. to the cubic foot, and cast iron weighs 450 lb. to the cubic foot. What per cent of the weight of cast iron is the weight of water? What per cent of the weight of water is the weight of cast iron?

9. A cubic foot of cork weighs about 15 lb. What per cent of the weight of water is the weight of cork?

10. What per cent of a bushel is 3 pk. 4 qt.?

11. What per cent of a mile is 352 yards?

12. A farmer sold $\frac{3}{5}$ of his farm. What per cent had he left? If he had 24 acres left, how many acres did he sell?

13. A football team won 3 games and lost 2. What per cent of its games did it win?

14. A lawyer charged \$ 68.75 for collecting \$ 550. What per cent did he charge?

15. A mixture of copper and zinc contains 21 parts of copper and 14 parts of zinc. What per cent of the mixture is copper?

16. There are 18 boys and 24 girls registered in a class. What per cent of the class is boys?

17. What per cent of a pound avoirdupois is a pound Troy?

18. A bankrupt owes \$ 7560 and has only \$ 4914. What per cent of his debts can he pay?

19. What per cent of a number is $\frac{2}{3}$ of it? $\frac{4}{5}$ of it? $\frac{5}{8}$ of it? $1\frac{1}{2}$ times the number? 2 times the number?

20. A train increases its speed from 40 miles an hour to 45 miles an hour. What is the rate of increase in per cent?

21. Mary is 12 years old and Frank is 16 years old. What per cent of Mary's age is the difference in their ages? What per cent of Frank's age is the difference in their ages?

22. If it takes a man 15 days to do a piece of work, what per cent of it should he do in 3 days? What per cent would remain to be done after he had worked 9 days?

Miscellaneous Exercises in General Applications of Percentage

168. 1. If 20 gal. of water are added to 250 gal. of milk, what per cent of the mixture is water?

2. A man's salary was increased $12\frac{1}{2}\%$, making it \$ 4500. What was his former salary?

3. The number of children in a town is 1104, which is 23 % of the whole population of the town. What is the population?

4. In playing tennis a boy won 60 % of his games, losing 8 games. How many games did he win?

5. After losing 25 % of his crop of wheat by a storm, a farmer harvested 960 bushels. How many bushels of wheat did he lose by the storm?

6. After my salary had been increased 30 % I received \$ 1560. How much was it increased?

7. After my salary had been increased \$ 200, I received \$ 3200. What per cent was it increased?

8. A man sold 104 sheep and had 221 left. What per cent of his flock of sheep did he sell?

9. There are 1080 pupils in a school, 45 % of whom are boys. How many girls in the school?

10. How many board feet of lumber will it take for a kitchen floor 12 ft. by 14 ft., if 25% is allowed for matching and waste?

11. In June there were 10 rainy days. What per cent of the days in June were pleasant?

12. After losing 12 marbles a boy has 3 left. What per cent of his marbles did he lose?

13. A house worth $ 5500 was damaged $ 1500 by fire. What was the per cent of damage?

14. In a school of 2500 pupils, 60 per cent are of foreign parentage. How many pupils have American-born parents?

15. A contractor agrees to do a piece of work for $ 5000. If his profit is $12\frac{1}{2}$ %, how much does he make on the contract?

16. From 4 bushels of seed 32 bushels of oats were harvested. What was the per cent of increase?

17. From an investment of 5¢ a boy makes 2¢. What is his per cent of profit?

18. If the bread made from a pound of flour weighs $33\frac{1}{3}$ % more than the flour, how many pounds of flour will it take for 60 lb. of bread?

19. A regiment of soldiers lost 400 men in battle out of 950 men in the regiment. What was the per cent of loss?

20. The average daily attendance in a certain school in September was 2215, in October 2318. What was the per cent of increase?

21. The area of a circle 2 ft. in diameter is what per cent of the area of a square 2 ft. on a side?

22. The diameter of a circle is what per cent of its circumference?

23. A heaped bushel contains what per cent of a stricken bushel?

CHAPTER X

SPECIAL APPLICATIONS OF PERCENTAGE

COMMISSION

169. Commission is the charge made by an agent or firm for doing business for another.

170. Commission is generally reckoned at a certain per cent on the amount of business done.

171. An agent or broker is a person employed to do business for another on commission.

172. A commission merchant is one who does the business of a merchant on commission, handling goods owned by other people.

173. The net proceeds on a sale of goods is the amount received by the owner after the commission and other charges, such as freight, storage, etc., have been deducted.

174. Commission, when reckoned at a given per cent, is reckoned on the **cost** when goods are bought, and on the **amount received** when goods are sold. It is reckoned on the **amount collected** when it is paid for collection.

Note. In all special applications of percentage, the key to the problem lies in the determination of the **base** upon which the **per cent** is reckoned.

Exercises and Applications in Commission

175. 1. Find the commission on a $750 sale at $2\frac{1}{2}\%$.

2. What is the commission for collecting 500, if the agent charges 5% commission?

3. What is the architect's commission on a church building costing $7500, if his rate of commission is $4\frac{1}{2}\%$?

4. What is the auctioneer's commission for selling $360 worth of property at auction, if he receives $7\frac{1}{2}\%$ on the sales?

5. A commercial traveler receives a salary of $25 per week, and a commission of 2% on his sales. How much does he earn in salary and commission in one week, if his sales amount to $1500?

6. A farmer sends some butter and eggs to a commission merchant, who sells them for $145, charging 5% commission. What is the net proceeds received by the farmer if the freight charges were $5.60?

7. A real estate dealer sells a house for $12500, charging $2\frac{1}{2}\%$ commission. What are the net proceeds of the sale?

8. In a city the man who takes orders from house to house for a grocer receives $2\frac{1}{2}\%$ commission on the orders taken. If he takes $950 of orders in a week, how much does he earn?

9. An agent sells 150 acres of land at $75 per acre, charging $2\frac{1}{2}\%$ commission. Find the agent's commission and the net proceeds of the sale.

10. A wheat broker sells 8000 bushels of wheat. What is his commission at $\frac{1}{8}¢$ a bushel?

11. How much did a coffee broker receive for purchasing 7500 lb. of coffee at 15¢ a pound, if he charges brokerage $\frac{1}{2}\%$?

12. If an agent receives $312.50 for selling $12500 worth of real estate, what is his rate of commission?

13. A broker buys 12000 bushels of wheat at $1.12 a bushel, charging $\frac{1}{2}\%$ brokerage, and then sells it at an advance of 3¢ a bushel, charging $\frac{1}{2}\%$ for selling. Find his total commission. Find the net profit to the firm for which the deal was made.

14. A book agent receives 20% commission on his sales. How much does he earn in one day, if he sells 6 books at $2.50 each?

15. A collector's commission at $1\frac{1}{2}\%$ amounts to $413.10. What sum did he collect?

16. An agent bought 5 city lots at $850 each, at a commission of $1\frac{1}{2}\%$, and charged for title insurance $50. What did the lots cost the purchaser?

17. An agent receives $510 with which to purchase goods after deducting his commission at 2%. What amount of goods could he purchase?

100 % of goods purchased = Goods purchased.
2 % of goods purchased = Commission.
102 % of goods purchased = $ 510.
$ 510 ÷ 1.02 = $ 500, value of goods. *Ans.*

18. Sent a check to my agent in Chicago to cover his commission and the cost of wheat purchased. If the check was for $ 1000, how much wheat should he buy, his commission being $\frac{1}{2}$ %?

Note. While the type of problems used in Nos. 17 and 18 are often used in school and in educational tests, they have little or no use in ordinary business.

COMMERCIAL DISCOUNT

176. Commercial or trade discount is an allowance or reduction from the list price or amount of a bill.

Manufacturers and wholesale dealers issue expensive catalogues and price lists, making the list price sufficiently high to cover all probable circumstances. They then make special terms to buyers by means of discounts. This enables them to change the net cost to meet change in market value, large purchases, prompt payment, etc., without making out new list prices or catalogues.

177. Discounts are reckoned on the amount from which the discount is made. When several discounts are named, as 15 %, 10 %, and 5 %, they are to be deducted one after the other, the first from the list, the second from the remainder, the third from

the second remainder, etc. It is customary, though not necessary, to deduct the discounts in the order named.

178. Net price is the price after all discounts have been made.

Net amount is the amount of the bill after discounts have been made.

Exercises and Applications in Commercial Discount

179. 1. Find the net price for arithmetics listed at 75 ¢, with discounts of 20 % and 10 %.

$$\$.75 \times .80 = \$.60.$$
$$\$.60 \times .90 = \$.54. \quad \textit{Ans.}$$

2. Find the net price of pencils listed at $5 a gross, with discounts at 20 % and 5 %.

3. Find the net cost of goods listed at $1200, the discounts being 25 %, 20 %, and 10 %.

4. Find the net cost of 280 lb. of rope, listed at $12\frac{1}{2}$ ¢ a pound, with discounts at 20 % and 5 %.

5. I bought goods listed at $750, discounts 20 %, 10 %, and 5 %. What was the net cost?

6. What is the difference on a bill of goods listed at $500, of a single discount of 25 % or discounts of 20 % and 5 %?

7. What single discount is equivalent to discounts of 15 % and 10 %?

$.85 \times .90 = .765$. $1.000 - .765 = .235$. $23\frac{1}{2}$ %. *Ans.*

8. What single discount is equivalent to discounts of 60 %, 40 %, 25 %, and 10 % ?

9. What is the net cash price of a carriage listed at $ 200, 15 % off and 5 % off for cash ?

10. Find the net cost of 8 plows listed at $ 20 each, discounts of 25 %, 10 %, and 5 % being allowed.

11. Which is better, and how much, for me to accept in buying a bill of goods listed at $ 300, a single discount of 45 % or discounts of 35 %, 10 %, and 5 % ?

12. If I get a discount of 20 % on school books and sell them at list price, how much do I make on goods listed at $ 75 ?

TAXES

180. A tax is money required from individuals for public purposes authorized by law.

181. Direct taxes are levied on the person or on the property which he possesses.

182. Indirect taxes are taxes upon articles of merchandise. They are called *duties* when levied on goods brought into the country from other countries, and *excises* or *internal revenue* when levied on goods made in this country.

Such taxes are called indirect taxes because while paid to the government by the person importing or manufacturing them, this tax is added to their sell-

ing price, and is finally paid, indirectly, by the one who buys the goods.

183. The tax on property is reckoned at a certain rate, usually expressed in mills of a dollar, the rate being calculated upon the assessed value of the property to be taxed.

184. Assessors are officers who estimate or assess the value of the property to be taxed.

185. Collectors are officers who collect or receive the taxes. They are paid either a certain per cent on the tax collected or a salary per year.

In cities, the officer who collects the tax is usually called the Receiver of Taxes.

Note. In large cities, the name of the owner of the property is not used, the lists being made out against the property, which is definitely described or located. If the owner fails to send the tax to the receiver of taxes at the proper time, the fine is charged against the property, and in due time the property is sold at public sale for taxes.

186. The tax rate is found by dividing the amount of tax to be raised by the total assessed valuation of the property subject to the tax.

Note. The books of the assessors are open to inspection by those interested, who may make application to have their assessments reduced, if they think them too high. After all such applications have been considered, the proper authorities make out the tax rate, which is usually announced in the local papers.

The tax due on each piece of property or from each person named in the books is then calculated, and the books delivered to the collector or receiver of taxes. The computations desirable for the general public to be able to make are limited, therefore, to finding the tax, when the rate and the assessed valuation are known.

187. Direct taxes are limited in this country at present to state and local taxes. The expenses of the United States government are met in other ways, largely by duties and internal revenue.

The following table shows the source of revenue for New York, and the amount raised in one year from the sources named:

NEW YORK STATE REVENUE, 1890–1891

From property tax,	$8,031,401
Corporation taxes,	1,535,875
Inheritance tax,	890,268
Interest on investments,	761,811
Fees and fines,	306,836
U. S. government,	109,903
Miscellaneous,	55,948
Total,	$11,692,042

Exercises and Applications of Direct Taxes

188. 1. The rate of taxation was recently announced as follows:

Rate per $100. City $1.40, School $.593.

Find the tax at those rates on property assessed for $5000.

2. The tax rate in the city of New York was recently announced as "1.55408 per centum." At that rate find the tax on property assessed for $5400.

3. The tax rate in the city of New York in 1900 was 2.32113 per centum. Find the tax on property assessed for $7500 at that rate.

4. The tax rate in a country place was announced recently as $2\frac{1}{2}$ mills on a dollar. Find the tax at that rate on property assessed for $2500.

5. The assessed valuation of property in a village was $3,200,000. What tax rate was necessary to raise $16,800 by taxes?

6. Find the tax on $6000 at $5\frac{1}{4}$ mills on a dollar.

7. The assessed valuation of the property in a school district is $546,000. Find the tax rate necessary in order to raise $3000 by taxes.

8. In a certain city the tax rate in 1908 was $1.43 on $100, and in 1909 it was 1.25 per cent. What was the difference in the tax to be paid by a man whose property was assessed each year for $3000?

9. Find the tax on $7500 at $1\frac{1}{2}$ per centum.

10. The assessed valuation of the property in a city is $7,950,000 and the estimated cost for schools in the city for one year is $13,900. Find the rate per cent of school tax.

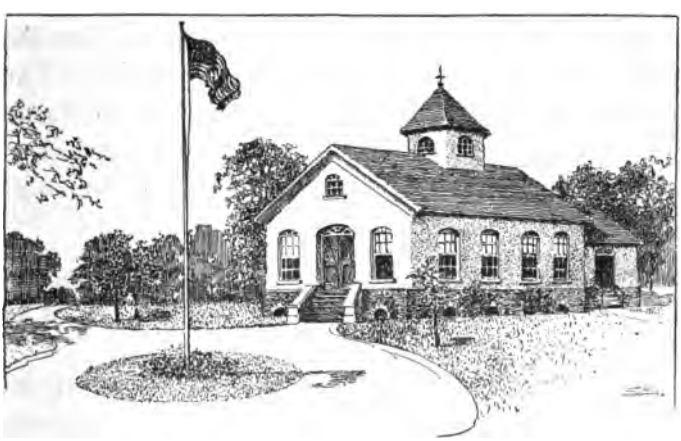
11. The tax rate is 3 mills on a dollar. What must I pay in taxes, if I am assessed $3000 for real estate and $2500 for personal property?

12. What tax will be paid by a man whose house is assessed for $5200, personal property for $2000, and who pays a poll tax of $1, the rate of taxation being $2\frac{1}{2}$ mills on a dollar?

13. My tax at 4 mills on a dollar was $24.60. What is my assessment?

14. At 5 mills on a dollar, what is the tax of a man who owns 150 acres of land worth $50 per acre, when property is assessed at $\frac{2}{3}$ of its value?

15. What rate per cent of tax must be levied on an assessment of $486,500 to raise $5800 by tax?



16. In a country school district the assessed valuation was $90,000. How much would it increase the taxes of a man whose property is assessed at $1000, if they should raise the salary of the teacher in the school $100 a year?

17. The taxes levied in a town are $24,000. If $1\frac{1}{2}\%$ is paid for collection of those actually collected, and 5% of the taxes cannot be collected, what are the net receipts in taxes?

INDIRECT TAXES

189. Duties or customs are taxes by the national government upon goods brought into this country from other countries.

190. Imported goods are those brought into the country. Exported goods are those sent out from the country. There is no tax on exports.

191. Ad valorem duty is duty charged on the cost of the goods. The duty on axes is 45%. The duty on musical instruments is 45%. These duties are ad valorem, 45% of their invoices, or appraised price if no invoice is presented, being collected as duty.

192. Specific duty is duty charged on the weight or quantity of goods imported without regard to their cost.

The duty on oranges is 1¢ a pound. The duty on onions is 40¢ a bushel. The duty on potatoes is 25¢ a bushel. These duties are specific.

Some articles pay both duties. For example: The duty on Axminster carpets is 60¢ a square yard specific and 40% ad valorem. The duty on revolvers is 75¢ each and 25%.

Many imported articles are on the free list, especially those which are needed in manufacture here, and cannot be produced in sufficient quantity in this country. Allspice, crude amber, animal hair, anthracite coal, gum arabic, India rubber, rattan, copper, silver, gold, ice, iodine, sulphur, etc., are admitted free of duty.

193. Duties or customs are collected at custom-houses which are situated at ports of entry established by law.

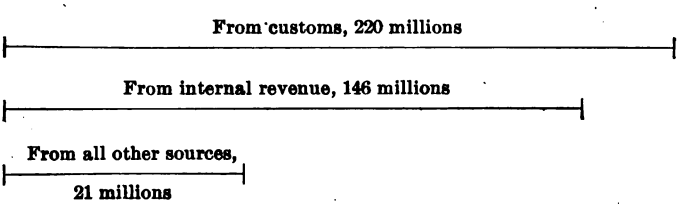
Every article brought into this country from a foreign port has to be inspected by government inspectors before it can be taken from the port. Travelers returning to the country have their baggage inspected, and if they have purchased abroad and brought back more than is allowed of any one thing or more than $100 in value for each person of all things, they must pay duty on the excess. They must also make oath that the goods which are allowed to pass free under the $100 provision are for personal use and not for sale.

194. Internal revenue is a tax upon the manufacture and sale of certain lines of goods, and upon the license to sell or trade in certain articles. It is levied chiefly against tobacco, whisky, etc.

This tax is collected by the state government in part at least when it is manufactured and sold entirely within the state, but by the national government when it passes to another state.

195. The United States Federal Revenue is raised largely from duties and internal revenue tax, as is shown in the following diagram based upon the revenue in a recent year.

DIAGRAM SHOWING SOURCE OF U. S. FEDERAL REVENUE



FOREIGN MONEY

As the duties on foreign goods when invoices are furnished with them is based upon their cost, as expressed in foreign money, it is desirable that some knowledge of the monetary system of the important countries that sell to us should be considered at this point.

EQUIVALENTS IN U.S. MONEY OF UNIT OF CURRENCY IN A FEW OF THE LEADING COUNTRIES

196. 1. England, 1 pound = $4.8665.
2. Germany, 1 mark = $.238.
3. France, Belgium, Switzerland, and Italy, 1 franc = $.193.

SUBDIVISIONS OF FOREIGN MONEY

197. England, 12 pence = 1 shilling.
20 shillings = 1 pound.
(The farthing is little used in English currency.)
Germany, 100 pfennigs = 1 mark.
France, 100 centimes = 1 franc.

In estimates, and for small change, we use 1 pound about \$ 5, 1 franc about 20¢, 1 mark about 25¢.

Exercises and Applications in Duties and Customs

198. 1. A shipment of silk from France, weighing 300 pounds and costing 2700 francs, was taxed at New York Customs, 40¢ a pound specific duty, and 15% ad valorem. How much duty was charged?

\$.40 × 300 = \$120, specific duty.
2700 francs = \$521.10.
\$ 521.10 × .15 = \$ 78.17, ad valorem duty.
\$ 120 + \$ 78.17 = \$198.17. *Ans.*

2. Find the duty on 200 yards of woolen goods, imported from England, and invoiced at 7*s.* 2*d.* per yard, and weighing $1\frac{1}{4}$ pounds to the yard, the duty being 44¢ per pound, and 55% specific.

3. A merchant imported musical instruments from Switzerland, invoiced at 3000 francs. What was the duty at 45%?

4. What is the duty on potatoes weighing 1200 lb., the duty being 25¢ a bushel specific? (Allow 60 lb. to the bushel.)

5. What is the duty on one ton of corn meal at 20¢ a bushel of 48 lb.?

6. What is the duty on 500 lb. of lemons, the rate being 1¢ a pound specific?

7. What is the duty on 10 gross of lead pencils costing $3 per gross in a foreign country, the rate of duty being 45¢ a gross specific, and 25% ad valorem?

8. What is the duty on ½ ton of butter, the rate being 6¢ a pound?

9. Bought cows in Holland at 200 florins each, and paid duty at 27½% ad valorem. Counting 40¢ to the florin, find the duty on each cow.

10. If an imported watch costs me $60 in New York, the duty being 40%, what was its list cost in Switzerland expressed in francs?

11. What is the duty on 25 lb. of perfumery that cost $2 a pound in Germany, when the rate is 60¢ a pound, and 45%?

12. When the duty on toy games is 35%, what indirect tax does a merchant pay who imports toy games that cost him 500 francs in Switzerland?

13. What is the duty on a musical clock bought in Geneva at 125 francs, the duty being 40% ad valorem?

14. A man buys books in London for 10 pounds and ships them to Boston. What duty will be charged, the rate being 25%?

15. What is the duty on 2 dozen English razors, costing 1 pound a dozen in London, the rate being $1.75 per dozen, and 20%?

16. What is the internal revenue tax on 5 gallons of alcohol, the tax being $1.10 per gallon?

17. What is the internal revenue tax on 500 lb. of oleomargarine manufactured in this country, the rate being 2¢ per pound?

18. What is the tax on 250 lb. of oleomargarine manufactured in a foreign country, the rate being 15¢ per pound?

19. The internal revenue tax on snuff and manufactured tobacco is 6¢ per pound. Find the tax on 250 lb. of snuff.

20. The internal revenue tax on cigars weighing over 3 lb. to the 1000 is $3 per 1000. Find the tax on a box of 100 cigars of this weight.

21. The internal revenue tax on cigars weighing 3 lb. or less per 1000 is $1 per 1000. Find the tax on a box of 50 cigars of this weight.

22. The internal revenue tax is $1.10 per gallon on whisky. Find the tax on a barrel of whisky containing 30 gallons.

INTEREST

199. Interest is the sum paid for the use of money.

200. The principal is the money for the use of which the interest is paid.

201. The rate of interest is expressed in per cent of the principal for the time of one year Interest

at 6 % means that a sum equal to 6 % of the principal is to be paid each year as interest.

202. The sum of the principal and the interest is often called the *amount*.

203. Time is always an important factor in computing interest. In ordinary calculations, the year is considered as 360 days, that the months may be considered as 30 days each.

204. The interest equals the product of the principal, the rate, and the time expressed in years.

The United States government uses accurate or exact interest, taking the actual number of days between dates and reckoning 365 days to the year. This method is used by some banks for large sums of money, and in legal calculations in courts.

Applications of Interest

205. 1. Find the interest on \$ 500 for 2 yr. at 5 %.

$\$500 \times .05 \times 2 = \$50$. *Ans.*

2. Find the interest on \$ 400 for 1 yr. 8 mo. at \$ 6 %.

1 yr. 8 mo. = 20 mo. or $\frac{20}{12}$ yr.

$\$400 \times .06 \times \frac{20}{12} = \$$ ——. *Ans.*

3. Find the interest on \$ 600 from Dec. 12, 1909, to Feb. 1, 1910, at 6 %.

Dec. 19 days
Jan. 31 days
Dec. 12 to Feb. 1, 50 days

$\$600 \times \frac{6}{100} \times \frac{50}{360} = \$$ ——. *Ans.*

4. Find the interest on $500 for 42 days at 6 %.

5. Find the interest on $9600 from May 16 to Aug. 14 at 5 %.

6. Find the interest on $500 for 1 yr. 5 mo. 12 da. at 5 %.

7. Find the interest on $750 for 7 mo. 20 da. at 6 %.

8. Find the interest on $350 for 1 yr. 3 mo. 18 da. at 6 %.

9. Find the interest on $850 for 2 yr. 7 mo. 24 da. at 4 %.

10. Find the interest at $4\frac{1}{2}$ % on $275 from Jan. 6 to Oct. 12.

11. Find the interest on $480 for 18 days at 6 %.

12. Find the interest on $3600 for 30 days at 4 %.

13. Find the exact interest on $3600 for 30 days at 4 %.

14. Find the exact interest on $7500 for 10 days at 6 %.

15. Find the exact interest on $25000 for 24 days at 5 %.

206. For convenience in reckoning time between dates, people who have a large number of examples in interest to work use a table like the one given on the following page:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Find the interest, by use of the table, from July 25 to Nov. 22, on \$1600 at 6 %.

Tracing the July line across to the November column, we find 123 in the table, which means that it is 123 days from July 25 to Nov. 25. To Nov. 22, it would, therefore, be 120 days.

$$\$1600 \times \tfrac{6}{100} \times \tfrac{120}{360} = \$32 \quad \textit{Ans.}$$

Without the table, we would find the time as follows:

From July 25	6 days more in July
to	31 days in Aug.
November 22.	30 days in Sept.
	31 days in Oct.
	22 days in Nov. to Nov. 22
	120 days

207. By the table for time, find the interest on the following:

1. $800 from Nov. 2 to Feb. 10 at 5%.

2. $1650 from May 4 to Dec. 1 at 4 %.

3. $ 1500 from June 6 to Apr. 5 at 6 %.

4. $ 800 from May 1 to Nov. 20 at 5 %.

5. $ 750 from Feb. 1 to Dec. 15 at 4 %.

6. $ 850 from Feb. 5 to Apr. 29, 1908, at 6 %. (Leap year.)

7. $ 2500 from Jan. 4, 1909, to June 15, 1909, at 5 %.

8. $ 3000 from May 6 to Dec. 12 at 6 %.

9. $ 1500 from Apr. 5 to Nov. 28 at $5\frac{1}{2}$ %.

10. $ 800 from Feb. 1, 1906, to Dec. 6, 1906, at 6 %.

11. $ 3500 from Sept. 12, 1908, to Apr. 10, 1909, at 6 %.

12. $ 4500 from May 4, 1908, to Dec. 1, 1908, at $5\frac{1}{2}$ %.

13. $ 1200 from Dec. 15, 1910, to May 12, 1911, at 6 %.

14. $ 750.75 from Aug. 1, 1906, to May 4, 1908, at 6 %.

15. $ 545.75 from June 6, 1908, to Dec. 5, 1909, at 5 %.

16. $ 325 from Jan. 2, 1900, to Aug. 12, 1901, at 5%.

208. As 6 % is a very common interest rate, the following method, called the "Six Per Cent Method," is often taught:

AT SIX PER CENT

.06 of the prin. = int. for 1 yr.
.005 of the prin. = int. for 1 mo.
.000$\frac{1}{6}$ of the prin. = int for 1 da.

To find the interest on \$ 500 for 1 yr. 8 mo. 18 da. at 6 %, we may apply the "Six Per Cent Method" as follows:

Int. 1 yr. = .06 of prin.
8 mo. = .04
18 da. = .003
Int. for given time = .103 of prin.
\$ 500 × .103 = \$ 51.50. *Ans.*

To find the interest at a different rate than 6 % by this method, we first find the interest at 6 %, then change it to the required per cent. For example, if the interest at 6 % is \$ 12, at 1 % it would be \$ 2, and at 5$\frac{1}{2}$ % it will be 5$\frac{1}{2}$ times \$ 2.

Exercises in Interest using Six Per Cent Method

209. 1. Find the interest at 4 % on \$ 300 for 1 yr. 9 mo. 18 da.

At 6 % 1 yr. = .06 of prin.
9 mo. = .945 of prin.
18 da. = .003 of prin.
Int. for given time = .108 of prin.
\$ 300 × .108 = \$ 32.40, int. at 6 %.
$\frac{4}{6}$ or $\frac{2}{3}$ of \$ 32.40 = \$ 21.60, int. at 4 %. *Ans.*

Note. Sometimes we change the decimal rate from that of six per cent to that of the required per cent, but as this often gives fractional results, it is better as a general plan to find the interest at 6%, and then change to the interest at the required per cent.

2. Find the interest on $560 for 1 yr. 5 mo. 15 da. at 6%.

3. Find the amount due on a note which is written for $50 and has interest due for 8 mo. at 5%.

4. What is the interest on $750 from March 4, 1908, to Jan. 6, 1909?

5. Find the interest from Oct. 3, 1907, to April 10, 1908, at 6% on $2500.

6. Find the interest on $850 from May 12, 1908, to April 1, 1910, at 4%.

7. What is the amount of interest due on a note for $600, the interest period extending from March 4 to Aug. 10, at 6%?

8. What should I receive in interest for the use of $75 from May 1 to Dec. 1, if money is worth 5%?

Note. While the Six Per Cent Method offers some advantages when in constant use so that one becomes expert in using it, for general practice where examples in interest do not occur very often, the safest and best method is the "cancellation" method, where the principal is multiplied by the rate expressed in fractional form, and the time expressed in years.

PROMISSORY NOTES

210. A promissory note is a written promise made by one person to pay another a definite sum of money at a definite time.

The **essentials** of a note are as follows:

1. It must name the amount of money to be paid.

2. It must state to whom the money is to be paid.

3. It must be signed by the person who promises to pay.

4. It must be dated and stated definitely when the money is to be paid, or the time for which it is loaned.

211. The following matters should be definitely understood by parties making or receiving notes:

1. Unless the place of payment is stated in the terms of the note, it is payable at the residence or place of business of the one who promises to pay.

2. When nothing is said about interest, no interest is to be paid until the note is due, after which, if the note is not paid, interest at the legal rate can be collected.

3. When the words "with interest" are used and the rate not stated, the legal rate of interest in the state in which the note is given is to be paid.

4. The form of the note is not important, so far as its value is concerned. If it contain the essen-

tials, it can be collected, no matter in what form these essentials are stated.

The following form is the one in common use:

$500 BROOKLYN, N.Y., MAY 27, 1909.

Three months after date, for value received, I promise to pay James Brown or order, Five Hundred Dollars, with interest at six per cent.

JOHN SMITH.

212. While the law concerning promissory notes is slightly different in different States, the following are generally true in all States:

1. The party to whom the note is to be paid may transfer it to another by indorsing it on its back, "Pay to the order of ----------" and writing his name. Some forms of notes are so worded that the one to whom they are first to be paid cannot transfer them by indorsement, but the common form of note can be transferred by proper indorsement.

2. The party receiving a note by indorsement may collect from the one indorsing it, if he cannot collect from the one who gave the original promise to pay in the note.

3. If the indorsement is made by simply writing the name across the back by the person to whom the money is to be paid, any person having the note legally in his possession may collect upon it.

4. Notes are often made by two or more persons interested in the loan. Such notes may read, "we promise to pay," "we jointly and severally promise to pay," or "I promise to pay," and be signed by more than one person. If the note reads "we promise to pay," all the signers must be sued together to collect the note in court. If either of the other forms named above are used, any one or all of the signers may be sued for payment. The payment by one releases all the other signers.

5. If a note fall due on a legal holiday, it is payable on the next business day in most States.

6. When the time given in the note is expressed in months, it falls due on the date found by adding the number of months to the date of the note without regard to "long" or "short" months.

7. When the time given in the note is expressed in days, the note falls due at the expiration of the days named.

8. A few States still allow "3 days' grace" in the payment of notes. That is, three days are added to the time expressed in the note before it can be collected by suit at law.

9. Notes are sometimes written, "On demand" "I promise to pay," etc. Such notes are payable on demand if no grace is allowed, and three days after demand in States which allow grace.

10. Sometimes a note is written without the words "or order" or "or bearer." In such cases the note is payable to the payee only and cannot be bought or sold, as they are not transferable by indorsement. They are called "non-negotiable notes."

11. The following terms are sometimes used:

The person who signs the note is called the maker.

The person to whom the note is made payable is called the payee.

The person who owns the note is called the holder.

12. To protest a note is to notify the indorsers in a legal way at the time of maturity of the note that it has not been paid. In most States the failure to protest a note when it is due releases the indorsers.

13. A higher rate of interest than that authorized by law is called usury. The penalty for usury varies in different States from the loss of the whole note to the loss of the interest, or the loss of the excess rate of interest.

213. Find the time of maturity and the amount due on the following:

1. Note for $ 300, given June 1, 1906, for 6 mo. at 6 %.

2. Note for $ 750, given May 10, 1909, for 9 mo. at 5 %.

3. Note for $ 150, given Aug. 4, 1907, for 2 yr. 6 mo. at 6 %.

4. Note for $400, given May 3, 1908, for 60 da. at 5%.

5. Note for $175, given Sept. 5, 1908, for 90 da. at 5%.

Find the amount paid in settlement of the following:

	Face	Date	Time	Rate	Settlement
1.	$150	Jan. 1, 1909	90 da.	5%	Maturity
2.	$300	May 2, 1907	On demand	6%	July 1, 1907
3.	$600	Aug. 4, 1907	2 yr.	5%	Maturity
4.	$500	Apr. 1, 1909	3 mo. (None, N.Y. State)		Sept. 1, 1909
5.	$350	May 10, 1908	1 yr.	6%	Sept. 1, 1909
6.	$750	Jan. 2, 1909	6 mo.	5%	Dec. 24, 1909
7.	$800	June 1, 1909	On demand	6%	Dec. 24, 1909

PARTIAL PAYMENTS

214. In country places, or in small business transactions, the maker is sometimes permitted to make payments on the note from time to time as he is able to do so. Such payments are credited on the back of the note, the date of the payment and the amount being paid being written across the back.

In large business dealings only such payments are permitted as are named in the written agreement. The application of partial payments is limited, therefore, to small transactions.

By the United States rule, if any payment does not equal the interest due at the time the payment is made, no change in the principal is made until

such time as the sum of the payments equals or exceeds the interest due at the time.

This rule is to protect the one making the payment, for should the amount due on the note be reckoned, and the payment deducted, if the payment were less than the interest due, the new principal would be greater than the former principal, and the maker would have to pay interest on a larger sum than if he had not made the payment.

Application of Partial Payments

215. 1. Find the amount due March 2, 1908, on a note for $ 200 given Nov. 26, 1906, with interest at 6 %, on which the following payments have been made:

Nov. 26, 1907, $ 50.
Jan. 2, 1908, $ 25.

From Nov. 26, 1906, to Nov. 26, 1907 = 1 yr.
Amount of $ 200 for 1 yr. at 6 % = $ 212
Payment, 50
New principal, $ 162
From Nov. 26 to Jan. 2 = 1 mo. 6 da.
Amount of $ 162 for 1 mo. 6 da. at 6 % = $ 162.97
Payment, 25.
New principal, $ 137.97
From Jan 2 to March 2 = 2 mo.
Amount of $ 137.97 for 2 mo. at 6 %
= $ 139.35. *Ans.*

2. Find the amount due at the time of settlement, Sept. 15, 1893, on a note for $912.75 at 6%, dated Aug. 3, 1890, on which the following payments have been made:

Jan. 1, 1892, $250.
May 4, 1893, $316.75.

3. A note for $1000, dated Aug. 1, 1900, at 6 % interest, had the following payments indorsed upon it:

Aug. 8, 1901, $200
Dec. 14, 1901, $200
Apr. 26, 1902, $200

How much was due Apr. 26, 1903?

4. A note given Jan. 1, 1890, for $700, interest at 6 %, has the following indorsements upon it:

May 6, 1890, $85
July 1, 1891, $40
Aug. 20, 1891, $100
Jan. 10, 1893, $350

How much was due on the note Sept. 30, 1894?

INSURANCE

216. An insurance company is a corporation which gives security against loss or damage under certain conditions.

217. The contract of insurance is called the policy. The face of the policy is the amount specified to be paid in case of complete loss.

218. The price paid for the insurance is called the premium. The premium on residences is usually quoted in terms for three years; on stores, factories, etc., in terms for one year. In life and accident insurance the premium is usually named in terms for one year at a certain sum for each $1000 of insurance.

General Notes on Insurance

219. 1. There are many kinds of insurance, such as insurance against loss by fire, loss of property at sea, loss by accident, loss by sickness, loss by death, loss by tornadoes, loss by burglars, injury to plate glass, etc. The most common forms for insurance of property are insurance against loss by fire, lightning, and storm; of persons, accident, sickness, and life insurance.

2. The rate of insurance is governed by the risk, which is very carefully estimated by the insurance companies, and different rates fixed for varied conditions or circumstances.

3. Aside from the regular insurance companies, there are many forms of mutual and association insurance. In these forms the losses are met by dues or by assessments. The principle upon which they are based is that the loss of one is shared by many.

4. Regular insurance companies have the same rate of insurance, which is fixed by general agree-

ment. In selecting a company, therefore, a person does not think of the rate, but gives careful attention to the financial standing of the company, the line of risks which they assume, and the form of contract which they issue in their policies.

5. There are so many kinds of insurance, and such a variety of policies in the same lines of insurance, that only the general calculations can be given here. When the terms of the policy are known, the computations to find the cost of the insurance are very simple.

Some Applications of Insurance

220. 1. What will it cost to insure a frame house for $4800, the rate being $4 on a thousand for three years?

2. Find the premium for insuring a brick house for $4800, at $2.50 per $1000 for 3 years.

3. What will it cost to insure my house, valued at $6000, and my furniture, valued at $1500, for $\frac{3}{4}$ of their value, the rate being $3.50 per $1000 for 3 years?

4. What is the annual premium for insurance on a store for $5000, the rate being 70¢ per $100 per year?

5. A man 30 years old, passing an examination for good health, has his life insured for $2500 in straight life insurance, the annual premiums being

$23.50 per $1000. How much premium will he pay in 5 years on this insurance?

6. A man 20 years of age insures his life on the 20-payment plan (premiums paid for 20 years, if he lives that long, then he can either draw out cash, or continue without further payment). The annual premiums are $26.90 per $1000. What does it cost him per year to carry this insurance?

7. A man takes out burglar insurance for $1500, the rate being $1\frac{1}{2}\%$ per annum. What does it cost him per year to carry this insurance?

8. A dealer insures a cargo of wheat valued at $18,000 for $\frac{3}{4}$ of its value, paying $15 per $1000 for the trip. What does the insurance cost him?

9. A farmer insures his growing wheat against damage by storm for $1000. What is the premium at $3\frac{1}{2}\%$?

10. The premium for insuring some property at 2% was $72. What was the face of the policy?

11. The rates for ordinary life policies are as follows:

Age 20 yr.	$18.50
25 yr.	$20.70
30 yr.	$23.50
35 yr.	$27.30
40 yr.	$32.20

Find the cost to a man who takes a policy for $2500 when he is 20 years old.

Find the cost of a $2500 policy taken at the other ages named in the rates given.

12. A man secured a health policy that would pay him $30 per week in case of sickness. This insurance cost him $50 per year. During 3 years he was sick three weeks in all, and drew his full insurance during that time. What was the net cost to him for his protection?

13. A man at the age of 40 years took out a 20-payment life insurance for $5000, at the annual rate of $40.60 per $1000. If he die after making 5 payments, how much more do his heirs receive than was paid to the company in premiums?

(The various investment phases of life insurance are too complicated and varied to be treated here.)

SUMMARY

A percentage of a number is the result obtained by taking a certain number of hundredths of it.

The term "per cent" means by the hundred.

The number on which the percentage is computed is called the base.

There are three general cases of percentage:

1. Finding a percentage of a number, as "Find 4% of $150."

2. Finding the number when the percentage and the rate per cent are given, as "$\dot{6}$% is $120; what is the number?" or, "After losing 10%, $810 remain; how much at first?" or, "After gaining 5%, a man had $210; how much had he at first?"

3. Finding what per cent one number is of another, as "6 is what per cent of 12?"

Special Applications of Percentage

Commission, when the charge made for doing business for another is at a certain per cent of the business done.

The key to commission is that the per cent expressions are computed on the business done for the firm paying the commission.

Trade discount is an allowance made from the list price or amount of a bill, stated in per cent reduction.

When several discounts are named, they are to be deducted one after the other.

Net price is the price after all discounts have been made.

A *tax* is money required from individuals for public purposes, duly authorized by law.

The tax on property is reckoned at a certain rate upon the assessed valuation of the property to be taxed.

Assessors are officers who estimate or assess the value of the property to be taxed.

The tax *rate* is computed by dividing the amount of tax to be raised by the total assessed value of the property subject to the tax.

Duties or *customs* are taxes on imported goods.

Ad valorem duty is reckoned on the cost.

Specific duty is reckoned on the quantity.

Duties are collected at custom houses which are situated at ports of entry established by law. The money collected is applied to expenses of the national government.

Internal revenue is a tax upon the manufacture and sale, or a license to trade in certain lines of goods, especially in tobacco and whisky.

Interest is the sum paid for the use of money.

The *principal* is the money for which the interest is paid.

The rate of interest is expressed in per cent of the principal for the time of one year. Time is a very important factor in all calculations of interest, the unit being one year.

Promissory notes are written promises made by one party to pay another a definite sum of money at a definite time.

The essentials of a promissory note are: the amount to be paid, to whom it is to be paid, when the note was given, definite time for payment, and signature of the party promising to pay.

Insurance is security against loss or damage under certain specified conditions.

The *policy* is the written contract between the parties.

The *premium* is the sum paid for the insurance.

The most common forms of insurance are fire, accident, and life.

CHAPTER XI

SPECIAL BUSINESS APPLICATIONS

INTRODUCTION TO SIMPLE ACCOUNTS

221. In keeping accounts, various forms are used, according to the extent and character of the business.

The forms shown on pp. 187 and 188 are for use in small business affairs, and for personal accounts with others.

The terms "Debtor" (Dr.) and "Creditor" (Cr.) are used in all forms, and have the same meaning.

(Title of Account)

			Dr.		Cr.	
Year Month	Day	Statement of items, etc.	$	¢	$	¢
		If kept against a person, the name of the person should be entered as "title of the account." If kept as a cash account, the word "Cash" is used as title. If used as an expense account, the word "Expense" is used as title, etc. Different pages are used for different accounts, and an index made to enable one to find a particular account quickly.				

222. The person or thing heading the account is debtor to what he or it **receives,** and credited by what he or it pays or gives.

223. Simple form for use in a small business, or by a laborer or mechanic or farmer, keeping a personal account with other people.

[SAMPLE PAGE OF DAYBOOK]

(Title of firm and date)

Page to which posted in ledger		(Name of person, etc.) (Dr. or Cr.) Statement of items.	Items	Total
		(Name of person, etc.) (Dr. or Cr.) Statement of items.	Cost by items	Total
		(New date.) (Name of person, etc.) (Dr. or Cr.) Statement of items.	Cost by items	Total

Notes. If this account was kept by a farmer to see what profit he made on a certain farm or field, the name of the farm or field would be entered as "title," and all things given to the farm or field, such as rent, labor, seed, fertilizer, etc., charged to it by entering them in the debit column. All things given by the farm, as crops, etc., would be credited to the farm by entering them in the credit column. The difference in the total debit and the total credit would give the loss or gain.

If kept as a cash account, the word "Cash" would be used as title, and cash on hand at the beginning of the account and all cash received would be entered as debit items. All cash paid out would be entered as credit items.

If kept by a grocer, the names of his different accounts would appear on different pages, and all things received by that which was used as "title" to the account would be entered as debit items

under that account. All things given or paid by that which was used as "title" to the accounts would be entered as credit items. In closing any account, the difference between the total debits and total credits would give the "balance," which may be entered on the smaller side and the accounts footed. The "balance" is then carried forward to the opposite column from which it was entered to close the account, and a new account is thus opened.

A laborer working for John Doe would enter the name John Doe in the title place of the account, and enter as debit items the work which he does and the cost of the material which he furnishes for Mr. Doe. He would enter as credit items all things received by him from Mr. Doe as they are given by that which forms the "title of the account."

224. In business where many accounts are kept, more than one book is used. The simplest form is where a daybook and a ledger are kept. Such books are suited for business too large for the first forms shown, and not large enough for the use of a full system of books and a regular bookkeeper.

The ruling for the daybook is the same as the ruling for the form shown where only one book is used. The chief differences in the daybook and in the one-book system are that in the daybook one account is written after another on the same page instead of turning to separate pages for separate accounts.

The columns for dollars and cents are also used differently, the first for items and the second for totals.

In "posting" the accounts to the ledger, the name is written on a page in the ledger, and then

the total is "posted" to its proper side, and an abstract made of the daybook statement of items.

When posted, the account in the daybook is checked as "posted" in various ways, one of which is to write the number of the page in the ledger to which it was posted.

[SAMPLE OF LEDGER PAGE]

DR. (Name or title of account) CR.

Year Mo.	Da.	Abstract from daybook. (This side for debit entries.)	D. B. page	$	¢	Year Mo.	Da.	Abstract from daybook items. (This side for credit entries.)	D. B. page	$	¢

To trace an account, we turn to the index in the ledger, which would show us the page for the account. Turning to the page, we find the account complete so far as totals are concerned. If we need to examine the items for which the totals are entered, we note in the page column, just before the column of dollars and cents in the ledger, what page the account was taken from in the daybook. We can then turn to that page in the daybook and find the full details of the account for that entry. Each other entry in the ledger can be traced in like manner. The page column in the ledger acts as an index to the account in the daybook.

In the daybook one man's account might be

written on many different pages, if he had entries on different dates, but they will all appear together in the ledger. The ledger is thus a book for collecting the various daybook entries into groups by accounts. The daybook can be written during the progress of business without turning pages, and the ledger posted from the daybook at convenient times. Many other books are kept, and many variations of the use of these books used. Each business has particular forms, which are easily learned after the general ideas are understood.

Introductory Applications of Simple Accounts

225. 1. As a carpenter, using the form shown for the single-book system, write up the following memoranda: June 1, 1909. Finished repairing house for James Ogden, having worked $3\frac{1}{2}$ days at \$3.50, and furnished 300 ft. pine lumber at \$25 per M, and 10 lb. nails at $4\frac{1}{2}$¢. June 2. Worked 5 hours for James Smith, repairing steps, at 45¢ an hour. June 5. Finished shingling veranda for Mrs. Sarah Brown, $2\frac{1}{2}$ days' labor at \$3.50. Furnished 1500 shingles at \$3.50 per M; 90 ft. pine lumber for cornice at \$25 per M; 15 lb. nails at $4\frac{1}{2}$¢ Mrs. Brown paid me cash on account \$10.

2. As a grocer, using the single-book system, write up the following memoranda of goods sold on account: June 1, 1909. Sold Frank Smith on a/c, 1 pkg. Quaker oats, 12¢; 3 lb. Hecker's buckwheat

at 5¢; 3½ lb. butter at 32¢. Sold Thomas Marsh on a/c, 1½ lb. cheese at 18¢; 5 lb. table salt at 6¢; 6 cakes ivory soap at 5¢; 1 lb. English breakfast tea at 60¢. Sold Leslie Johnson on a/c, 1½ lb. coffee at 32¢; 12-lb. sack flour for 50¢; ½ gal. molasses at 65¢; 1 doz. pkg. Korn Kinks for 55¢.

3. As a farmer, using the single-book system, write up the cash account for the following: June 1, 1909. Cash on hand $25. Sold for cash 5 bu. potatoes at 75¢; paid for feed, $1.80. June 2. Paid for shoeing horses, $1.50; bought for cash 1 sack flour, $1.25; 1 pair shoes for self, $2. June 3. Sold 3 calves at $7.50 each; paid James Smith for work in corn field, $3.50; paid for repairs to wagon, $3. June 5. Sold 25 bu. oats for cash at 45¢ a bushel. Balance the account and see how much cash the farmer should have at the close of the week.

4. As a general merchant in a small business, write up the following memoranda, using the day-book and the ledger: Abstract of a few accounts at the opening of business June 1, 1909: Cash on hand, $450. Andrew Jackson owes me $15; George Kelly owes me $25.45; Charles Gray owes me $10.75; William Smith owes me $18.60; I owe James Haig $30 and Edward Stanton $62.50. June 1, 1909. Sold for cash goods amounting to $35.75. Sold on account, to Andrew Jackson, 1

suit clothes $15; to William Smith, 1 hat, $1.75, and 1 pair shoes, $2; to James Haig, 1 pair suspenders, 50¢, and 1 pair trousers, $3.50. June 2. Sold for cash goods amounting to $30.25. Paid cash for goods, $50. Sold on account, to Edward Stanton, 1 coat, $6, and 1 tie, 35¢; to George Kelly, 1 suit clothes, $18.50. June 3. Sold for cash goods amounting to $25.75. Paid bill from John Doe & Co., in cash, $35.50. Sold on account, to Andrew Jackson, 1 pair shoes, $3.50; to Charles Gray, 1 hat for $1.75 and 1 tie for 50¢; to James Haig, 1 hat for $2. June 4. Sold for cash goods amounting to $35.75. Paid rent of store for June in cash $35. Sold on account, to Andrew Jackson, 1 white shirt $1.25; to Charles Gray, 1 coat and vest for $12.50; to George Kelly, 1 pair shoes, $3. William Smith pays me cash on account $10. Write up these abstracts and see what each man owes and what amount of cash the merchant has on hand.

BILLS AND RECEIPTS

226. A *bill* is a creditor's written statement of items in his account against a debtor. When sent with goods, bills are often called invoices.

227. A *statement* is a written summary of an account. It is like a bill, except that it does not usually give the full details of the account by items.

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228. A *receipt* is a written acknowledgment of something received. Receipts are usually for money received, but may be given for anything of value.

229. In selling a list of goods and in presenting a claim for payment for services rendered or material furnished, a bill is made which shows the date, the person who is debit, and the person and place of business of the one making the claim, and the full particulars as to each item of the account.

When the bill is paid, it is receipted by the one who receives payment. A bill is receipted by writing the words, "Received payment," or the word "Paid" on its face and signing the name by the one who receives payment. The bill is then returned to the one who paid, and kept by him as evidence that it has been paid.

When bills are sent with goods, they are sometimes marked "C. O. D.," which means that the bill must be paid when the goods are delivered. They often have the terms of payment written upon them, as "ten days," etc., which gives the time of credit allowed on the bill. Under this head of terms, the discounts, if any, are also quoted. (See Commercial discount.)

When a person makes payment where no bill has been rendered, a written receipt should be given which may be kept by him as evidence that the debt has been paid.

When a debt is paid by check, no receipt is necessary as evidence that such a sum of money has been paid, as the check will be returned to the maker, after being cashed and canceled by the bank, and may be held by him as a voucher. The check will not show the particulars of the debt, however, so that a receipt or a receipted bill is usually kept as well as the voucher.

Application of Bills and Receipts

230. 1. James Smith, carpenter, works 3 days at $3.50 per day for James Brown, repairing his house. He furnishes lumber worth $5 and nails and screws worth 50¢. Make out the bill which Mr. Smith should present.

COVENTRY, N.Y., Jan. 5, 1909.

Mr. James Brown,
To James Smith, Dr.

To 3 days' labor at $3.50,	$10.50	
lumber,	5.00	
nails and screws,	.50	$16.00

(Ruled forms are commonly used, but are not necessary, as the bill may be rendered as written above, when the ruled form is not at hand.)

Note. In cities, where the people are not well known to each other and in all cases where the bill is to be sent by mail, the bill should give the business address of both parties, — as Mr. James Brown, 540 Fulton St., Brooklyn, To James Smith, 345 Adams St., Brooklyn, etc.

2. The Brooklyn Furniture Company of 440 Fulton St., Brooklyn, June 1, 1909, sells for cash to Mr. Thomas Marsh, 430 Bedford Ave., Brooklyn, 6 chairs at $3; 2 armchairs at $6.25; 1 rocking chair at $5.50, and 1 table for $20. Fill in the blanks in the following form to make the proper bill for this sale:

BROOKLYN FURNITURE COMPANY,

440 Fulton St., Brooklyn.

Terms net cash.

BROOKLYN, N.Y. ----------190----

Sold to ----------

---

3. Rule a form and make out a receipted bill for the following: Mr. Charles Haines sold to

Mr. Frank Smith 25 bags of oats at 64¢; 30 bags of corn at 85¢; 12 bags of feed at 65¢, receiving payment in cash.

4. Mr. Samuel Adams of 47 Tenth Ave., Mount Vernon, N.Y., pays Mr. Thomas Brown for the May rent of the store at 47 Fourth Ave., Mount Vernon, $150, making the payment on June 2, 1909. Write the receipt in a ruled copy of the following form:

MOUNT VERNON, N.Y. ..... 190....

Received from .....

..... dollars

for .....

$ ....

5. Suppose that you have sold to Mr. John White 4 pear trees at $1.75; 9 geraniums at 12¢; 12 tomato plants at 5¢. Make out the bill which you would send with the goods to Mr. White.

6. Suppose that you have worked for Mr. Samuel Brown as follows: June 5, 1909, 5 hours at 20¢; June 8, 10 hours at 20¢; June 12, 6 hours at 20¢; June 15, 8 hours at 20¢. Make out the bill which you would present when asking for payment.

7. Morse and Hayes, Grocers, Greene, N.Y., find by their books that Mr. Joseph Juliand owes them for groceries as follows: Aug. 6, 1908, 1 gal.

oil, 15¢; 1 yeast cake 2¢; 1½ lb. butter at 32¢. Aug. 8, 6 lb. sugar at 5¢; 1 sack flour $1.75; 1 bag salt 25¢. Aug. 13, 3½ lb. cheese at 16¢; 2 lb. coffee at 38¢; 6 bars soap at 5¢; ½ lb. tea at 60¢. Make out the bill that they would send in asking payment for the debt.

8. Mr. James Brown owes you $75 for some hay which he bought last winter. On June 1, 1909, he calls and pays you $25 on account, not bringing the bill. Write out a receipt for the payment.

9. Sandford Coal Company, 2d Ave. and 3d St., Brooklyn, N.Y., deliver coal to James B. Haig, 430 Putnam Ave., Brooklyn, N.Y., sending a "C. O. D." bill with the driver. They deliver 5 tons Lehigh coal, nut size, at $6.50 per ton, and 8 tons Scranton coal, egg size, at $6.60. Make out the bill as they would hand it to their driver.

10. On June 5, 1909, The Macmillan Company sent from their New York City office to R. S. Bowne & Co., Albany, N.Y., the following books: 50 grammars at 40¢, 100 arithmetics at 60¢, 50 geographies at $1.25, and 45 histories at 90¢. They allow a discount of 10 % to this firm, if payment is made in cash. Make out the bill.

Note. As printed forms are so generally used in making bills and receipts, further study in this subject may better be made by the use of such forms, and in the study of local conditions suited to the probable needs of the class.

BANKING

231. When a person (or a firm) has a little money ahead, he usually deposits it in a bank for safe keeping, and for convenience of doing business. If he wishes to use a portion of it from time to time and to make new deposits, he does business with a Bank of Deposit and Discount. If he can leave it for some time and wishes it to draw interest, he does business with a Savings Bank.

232. Commercial, national, state, private banks, etc., receive money for safe keeping, and loan money in various ways. They also render services in many ways to business men. As a rule they do not pay interest on the money deposited with them.

233. Savings banks pay the depositors interest on their deposits left during the full interest term. They pay interest on the smallest amount which was in their charge during any portion of the term. A man having $500 in a Savings Bank for two of the six months' interest term, and having $250 in during the remainder of the time, would receive interest on the $250.

234. Trust companies pay low rates of interest on monthly balances and also pay out on check by the depositors. They combine the business of the deposit and discount banks and that of the savings bank, giving something of the advantages of each, but not the full advantage of either.

235. In opening an account at a bank of deposit and discount, a person takes his money there, makes out a deposit slip on the printed blank furnished by the bank, gives references to his personal reliability, if unknown to the bank officials, and leaves his signature as a matter of reference. He receives a blank check book from the bank, and a pass book in which is entered by the receiving teller the amount which he has deposited. He may then pay bills, etc., in his business by writing checks for the amounts, checking against his bank account as long as it lasts. When he makes further deposits, he presents the pass book each time, and the receiving teller writes in the amount received by the bank. From time to time he leaves this pass book to be "written up," and the bank clerks enter upon it the account of all checks paid by them for him, and calculate his balance. They also return to him the canceled checks which they have cashed, that he may keep them as vouchers or receipts for payments made.

236. In the check book there is printed with each blank form for a check a blank form, called a "stub" which remains in the book after the check is removed, and upon which he enters a memoranda of the date, person to whom paid, for what paid, and the amount of the check. He also calculates his balance remaining in the bank on each stub, that he may not overdraw his account.

237. The person receiving the check must write his name across its back before he can use it. This is called indorsing it. He may write "Pay to the order of James Brown" and sign his name and send the check to Mr. Brown, who can then use it by making further indorsements upon it. To have a check cashed at a bank, the person presenting it must indorse it, and if unknown to the paying teller, prove that he is the person to whom the check should be paid.

238. The advantages of doing business with a bank are many to the business man, and to the person who does a small business it gives security for his money, receipts for his payments, and enables him to make payments by sending checks by mail. Checks from large central banks like the prominent banks of New York City are cashed for the full amount for a considerable distance from the city. On checks from small banks a small sum for collection is charged.

239. If a man wishes to leave his money with a bank for a short time as a matter of security from loss, and does not expect to check against it, he takes a *certificate of deposit.* This is like a receipt, and must be presented when payment is made by the banks. When certificate of deposit is taken, checks cannot be written against the bank account.

240. Any one receiving a check should use it at once, as the law offers no protection if he hold it beyond a reasonable time. If he does not wish to draw the money, or wishes to send the check to another party, he may have it certified by the bank of issue. Banks certify checks by writing the word "certified" in red ink across the face, and then set aside a sufficient sum from the account of the man who signed the check to insure payment. Some business men require that all checks sent them shall be certified.

241. The various banking forms, such as deposit slips, checks and stubs, and pass books, should be examined and studied from those in local use. The following forms are those of a prominent New York City bank.

(*a*) **Deposit slip.**

Deposited by

---

in

The Importers & Traders National Bank
of New York.

New York, ----------191

Specie		
Bills		
Checks		

(*b*) Check and stub.

Balance, ... $_____

*Deposit,*_____ $_____

Total, ... $_____

Drawn, ... $_____

Balance forward, $_____

*No.*_____*Date,*_____ _____$_____

*Favor of*_____

No._____ New York,_____190

THE

Importers and Traders National Bank OF NEW YORK.

THROUGH THE NEW YORK CLEARING HOUSE ASSOCIATION.

Pay to the order of_____

_____Dollars,

$_____ _____

(The pass book cannot be conveniently shown here. It is a small account book, one page for debit and one page for credit with printed headings, and ruled for dates and dollars and cents on each page. The debit and credit pages are opposite each other.)

Savings Banks

242. Savings banks receive money and invest it in business, paying the depositors interest on sums remaining with them for their interest term, usually three months and six months.

The interest periods of most savings banks end

Jan. 1 and July 1, and interest is allowed on all sums which have been on deposit for six months or for three months previous to these dates.

243. A person depositing money in a savings bank makes out the deposit slip, signs the agreement to the bank regulations, gives personal information about himself for the purpose of future identification, and leaves his legal signature.

The bank gives him a bank book, in which is printed the rules and regulations of the bank, and full information about its interest periods, etc., and in which the receiving teller enters the amount of his deposit. No check book is furnished, as a person cannot draw a check against an account in a savings bank.

Payments are made by the bank only when the bank book is presented with the order for the payment. When the payment is made, the amount is entered in the bank book, so that this book at all times shows the amount on deposit, unless there may be some back interest which has not been credited.

244. At the close of each interest period the bank computes the interest due on each of its accounts, and credits the depositor with the same, whether his book is presented or not. For the next interest period he draws interest on this interest which has been thus credited.

Savings banks have recently been paying 4 %, so that $ 1000 remaining a full year in a savings bank would amount, at 4 %, to $ 1040.40, as follows:

Int. on $ 1000 for 6 mo. at 4 % = $ 20.
Amt. of $ 1000 for 6 mo. at 4 % = $ 1020.
Int. on $ 1020 for 6 mo. at 4 % = $ 20.40.
Amt. of $ 1020 for 6 mo. at 4 % = $ 1040.40.

Applications of Banking

245. 1. Make out a deposit slip for the present date, and in your name, for specie $ 5; bills, $ 50; checks $ 150, $ 75, and $ 60.

2. Draw a check and make the proper stub memoranda for the payment for last month's rent on your house, making check in favor of the owner, James S. Brown, and for $ 35. Draw the check on the First National Bank of Boston, using present date.

3. Draw a check with present date in favor of John Doe for $ 75, in payment for a wagon. Draw the check on the Brooklyn Trust Company.

4. A man deposited in the Emigrant Industrial Savings Bank of New York as follows: Jan. 2, 1908, $ 500. April 1, 1908, $ 250. If he does not withdraw any money, and the bank pays 4 % interest, how much would be to his credit on Jan. 1, 1909?

5. You receive a check made by Richard Smith to the order of yourself, and you wish to transfer it to Frank Brown. The check was drawn on the Brooklyn Trust Company for $75, and dated May 10, 1909. Write the check and then make the proper indorsement upon it, that it may be collected by Mr. Brown.

6. A contractor in New York City, in sending in a sealed bid for a contract, has to inclose a certified check for $500 as security that he will carry out the contract if awarded to him.

Write this check in favor of the Board of Education, on the Hanover National Bank, dating it June 10, 1909. Write the certification of the check across its face as it would be made by the bank.

7. Draw a check in favor of the American Book Company, on the Bank of Commerce, for $34.75, using present date and your name. Make out the stub memoranda to show that the payment was for a bill of books.

8. On Jan. 1, 1909, James Brown had a balance in the bank of $150.24. Since that date the stubs in his bank book show the following: Jan. 2, drew check No. 785, $30; deposited check for $50; Jan. 6. drew check No. 786, $8.75, No. 787, $15.60; Jan. 9, drew check No. 788, $75; Jan. 12, deposited check for $75.50; Jan. 16, drew check No. 789 for $85.25. Find his balance Jan. 18.

9. How much more income would a man receive in two years from $500 loaned at 5% interest than if he had kept it in the savings bank which paid 4% during the entire time?

10. Write such a check as you would present at the bank when you wished to draw out $50 for personal use, using your name and the present date. (Draw the check to the order of "self" or "cash.")

BANK DISCOUNT

246. Bank discount is the interest charged for advancing the payment of money. It is the same as simple interest, only that it is paid in advance.

When a man borrows money from a bank, it takes out the interest for the time which he is to have the use of the money, and usually keeps some form of security for payment, called collateral security, which the bank may sell for payment of the loan, if the money is not paid.

Banks often loan money without collateral security, if the names of two responsible persons are given on the note as security for its payment.

In notes for money borrowed direct from the bank, nothing is said about interest, as that is covered by the bank discount.

Notes for money borrowed direct from the bank are usually written for 30, 60, or 90 days, and then renewed if arrangement is made for a longer time.

Notes from other persons, and various forms of negotiable paper, are often discounted at the bank, if the person to whom the money is to be paid desires to use it before the maturity of the negotiable paper.

247. Bank discount is expressed in rate per cent, and is always reckoned upon the amount due at the time of the maturity of the paper discounted.

248. The term of discount is the time from the date of discount to the date of the maturity of the paper discounted.

249. The amount of the note at its maturity, less the bank discount, is called the proceeds. It is the sum which the bank will advance for the note.

When the note discounted does not bear interest, its face and its amount at maturity are the same. When the note bears interest, the amount at maturity is its face plus this interest.

Bank discount is computed upon the maturity value.

Applications of Bank Discount

250. Find the proceeds:

1. On \$75 discounted for 30 days at 6%.
2. On \$150 discounted for 90 days at 6%.
3. On \$500 discounted for 60 days at 6%.
4. On \$300 discounted for 30 days at 5%.

5. On \$750 discounted for 60 days at $5\frac{1}{2}\%$.

6. On \$1200 discounted for 90 days at $5\frac{1}{2}\%$.

7. On \$2000 discounted for 15 days at 6%.

8. On \$3500 discounted for 10 days at 6%.

9. On \$5000 discounted for 20 days at 6%.

10. On \$3750 discounted for 6 days at 6%.

251. Find the maturity, the term of discount, the discount, and the proceeds:

1. On a 60-day note for \$300 dated Oct. 12, without interest, and discounted Nov. 1 at 6%.

Oct. 12 + 60 days = Dec. 11, date of maturity.
Nov. 1 to Dec. 11 = 40 days, term of discount.
$300 \times \frac{6}{100} \times \frac{40}{360} = \$2$, bank discount.
\$300 − \$2 = \$298, proceeds.

2. On a 45-day note for \$6000 with interest at 4%, dated Dec. 28, and discounted Jan. 2 at 5%.

3. On a note for \$720 for 3 months at 6% interest, dated July 17, and discounted Sept. 1 at 5%.

4. On a 90-day note for \$500 with interest at 6%, dated May 4, and discounted June 8 at 6%.

5. On a 9-month note for \$450 with interest at 6%, dated Oct. 15, 1906, and discounted Feb. 25, 1907, at 6%.

252. Find the proceeds:

	Face	Date	Due	Interest	Date of Discount	Rate of Discount
1.	$150	July 1	Aug. 1	6%	July 15	6%
2.	$500	Feb. 1	Apr. 1	none	Feb. 1	6%
3.	$750	Aug. 1	Dec. 1	5%	Oct. 1	6%
4.	$1000	Jan. 1	June 1	none	Apr. 1	6%
5.	$300	May 10	Aug. 10	4%	June 1	6%
6.	$7500	Apr. 1	June 1	6%	Apr. 1	6%
7.	$600	March 5	June 5	none	Apr. 1	6%
8.	$500	July 1	Nov. 1	none	July 1	6%
9.	$350	Feb. 5	May 1	4%	March 1	$5\frac{1}{2}$%
10.	$450	Jan. 1	July 1	6%	Apr. 1	6%

Some States include the day of discount as well as the day of payment, thus making the term of discount 1 day longer.

TRUE DISCOUNT

253. True discount is the difference between the amount of a bill due at some future time and its present worth.

254. The present worth of a bill or debt is the sum which, put at simple interest, will amount to the debt when it is due.

Note. True discount is less than bank discount by the interest on the true discount for the time. True discount is little used in business at the present time.

Applications of True Discount

255. 1. Find the present worth and the true discount of \$600 due in 5 months, money being worth 6 %.

The amount of \$1 for 5 mo. at 6 % is \$1.025.

\$600 ÷ \$1.025 = \$585.37, present worth.
\$600 − \$585.37 = \$14.63, true discount.

PRINCIPLE. To find the present worth by true discount, divide the given amount by the amount of \$1 for the given time and rate.

2. Find the bank discount and the proceeds for \$600 due in 5 mo., discount at 6 %. (Compare this answer with that of Ex. 1.)

3. Find the present worth of \$590 due in 1 year 6 months, the current rate of interest being 6 %.

4. If money is worth 6 %, find the difference between the present worth and the bank proceeds of a note for \$750 due in one year without interest.

5. A man buys flour for \$2500, payment to be made in 6 mo. What is the present worth of the debt, money being worth 5 %?

6. What sum must I put at interest at 6 % to amount to \$500 in 2 years?

7. A man owes me \$80 due in 60 days without interest. How much more will I receive if I have the note discounted by true discount than if I have

it discounted by bank discount, the rate being 6 % in each case?

8. A man pays his rent, at the rate of $600 per year, quarterly in advance. How much more does it cost him for the year than if he had paid it at the end of each quarter, money being worth 6 % ?

9. A man offers to sell a horse for $200 cash or $225 due in 6 mo. Which is the better offer for the purchaser, money being worth 6 % at true discount?

10. I buy a horse for $180 and sell him on a note due in 90 days without interest, for $200. What is my net gain at the time, money being worth 6 % at true discount?

STOCKS AND BONDS

256. A corporation or company is formed by a number of persons when the business requires a large capital.

It has certain powers and privileges granted it by law, and is subject to certain limitations. It can then do business as if it were a single person.

257. The declared capital when it is organized is called its capital stock, which is usually divided into shares of $100 each which are owned by the stockholders.

258. Corporations are managed by officers or directors who are elected by the stockholders.

259. Certificates of stock are given to each stockholder, showing the number of shares to which he is entitled and the par value of each share. These certificates may be sold to another person by the stockholder at any time and for any price that he can get. Transfer is made so that the purchaser has his name entered upon the books of the company, and thus becomes entitled to vote at its meetings.

260. Dividends are profits divided among the stockholders.

261. Many companies issue both preferred and common stock. Preferred stock is generally entitled to a fixed rate of dividend before the common stock can share in the profits.

262. Par value means the face value of the stock.

Above par or at premium means a value above the face.

Below par or discount means a value below the face.

263. Bonds are formal written obligations under seal securing the payment of certain sums of money to the purchasers at a specified time with interest at a fixed rate.

264. Government bonds are issued by the United States, by states, cities, counties, etc., when duly

authorized by the proper officials for the various expenses of government.

Stock companies also issue bonds for the purpose of effecting loans and thus raising money.

265. A stock broker is an agent who buys and sells stocks and bonds for others. His commission is called brokerage.

266. Market value is the price at which stocks or bonds are selling in the market. Bonds pay interest at their face value, hence the market value does not change the interest which they yield.

Stocks are bought for speculation, largely, as their market value changes widely from time to time. Bonds are more often bought for investment.

The daily papers give the market price for the leading lines of stocks and bonds.

Exercises and Applications

267. 1. What is the cost of 75 shares of American Sugar Stock at $132\frac{3}{8}$, brokerage $\frac{1}{8}$ %?

1 share costs $\$132\frac{3}{8} + \$\frac{1}{8}$ or $\$132.50$.

75 shares cost 75 times $\$132.50$, or $\$9937.50$. *Ans.*

2. How much will be received by the owner for 90 shares of railroad stock at $132\frac{1}{2}$, brokerage $\frac{1}{8}$ %?

$\$132.50 - \$.125 = \$132.375$, net for each share.

$\$132.375 \times 90 = \$11913.75$. *Ans.*

3. My broker bought for me 50 shares of Erie R. R. stock at $36\frac{1}{2}$, brokerage $\frac{1}{8}$. Find the total cost.

4. I invested \$4535 in U. S. 4's at $113\frac{1}{4}$, brokerage $\frac{1}{8}\%$. What does the government pay me annually in interest?

5. If 250 shares of stock are bought at $12\frac{1}{4}$ and sold at $13\frac{1}{8}$, brokerage $\frac{1}{8}\%$ in each case, what is the net profit to the dealer?

6. One morning a man bought 1000 shares U. S. steel preferred at $125\frac{3}{4}$ and sold it a few hours later at 128. What is his net profit, after paying brokerage $\frac{1}{8}\%$ both ways?

7. How much must be paid, including brokerage, for 250 shares of N. Y. C. R. R. stock at 134?

8. What per cent income shall I receive upon my investment if I buy 4 % bonds at 80? (Income on \$80 is \$4.)

9. Bonds selling at 90 pay 4 % dividends. What per cent on the investment may be made by the purchase of these bonds?

10. I hold 100 shares of B. R. T. stock, which was quoted at 76 Jan. 4 and at 79 Jan. 6. How much has this advance in the market increased the value of my holdings in this stock?

PROFIT AND LOSS

268. The profit and loss in business transactions is usually named in terms of per cent on the money invested.

The per cent of profit or loss is always reckoned

upon the cost. The cost is the **key** to the solution of most problems in profit and loss.

Exercises and Applications

269. 1. What is gained by selling a hat that cost \$3 at a profit of 20%?

2. Sold a knife for 60¢ that cost 45¢. What was the rate of gain?

3. Flour sold at \$6 per barrel gives a profit of 20%. What is the cost?

4. A merchant selling goods at a profit of 10% makes \$75. Find the cost and the selling price of the goods.

5. A merchant sold hats at a bargain sale for \$2.99, gaining 15% on each hat. Find the cost of the hats and the net profit on the sale of 20 hats.

6. A boy buys apples at the rate of 3 for 5¢ and sells them at 2¢ each. What is the rate of profit?

7. A dealer marked the selling price of his goods 25% above their cost. What per cent of their cost would he make or lose if he should sell them at a special sale 25% below the marked price?

8. What is the rate of profit on pencils bought at \$4.50 per gross and sold at 5¢ apiece?

9. Pens bought at 40¢ a gross are sold at 2 for 1¢. What is the per cent of profit?

10. Chairs listed at $2.40 are bought at discounts of 20% and 5%, and sold at 10% above their list price. What is the per cent of profit?

11. If 8% is lost by selling a chair for $1.15, what was its cost?

12. By selling a wagon for $90, a dealer made 25% profit. Find the cost.

13. A merchant sells his goods at 20% profit, but loses 5% of his sales for bad debts. What is his net rate of profit?

14. Bought gloves in Paris at 40 francs a dozen, paid a duty of $3 a dozen, and then sold them in Boston at $1.25 a pair. What was my net profit on 6 dozen pairs of gloves? What was the per cent of net profit on the cost in Boston?

15. Bought silk umbrellas in London at £5 a dozen, paid the duty of 50%, and sold them in New York at $4 each. What was my per cent of profit on the cost in New York?

16. Bought leather pocket-books in Germany at 1 mark each, paid the duty of 35%, and sold them at 50¢ each. What was the per cent of profit on the cost in this country? (1 mark = 23.8 cents.)

17. Bought scissors in England at 10 shillings a dozen, paid the duty of 75¢ per dozen specific and 25% ad valorem, and sold them at 50¢ a pair. What was the per cent of profit on the cost in this country, counting 1 shilling as $.243?

18. Sold a bill of goods costing $200, at a profit of 20 %, receiving in payment a note without interest for 90 days which I had discounted at 6 %. What was my net profit on the goods in cash value?

19. Sold some goods at 30 % profit, but was unable to collect the full debt, and compromised on 80¢ on a dollar. What was my net per cent of profit or loss on the transaction?

20. I sold a horse for $240 and thereby lost 20 %. What selling price would have given me a profit of 20 %?

PARTNERSHIP

270. When two or more persons enter upon some business, and have formal agreement about their shares of profit and loss, they are said to have formed a partnership, and are called partners.

271. The partners are sometimes referred to collectively as a "house," but are more often called a "firm."

272. Partnerships are usually formed by written contract, but may be formed by oral agreement or by the persons simply acting together in a common business transaction.

273. In the regular business transactions of the firm, the act of one partner is binding upon all.

274. The investment of a partner is called his capital, which may be money, real estate, stock,

labor, experience, "good will," or anything of value to the business.

Exercises and Applications

275. 1. Three men engage in business as partners; A invests \$8000, B \$12000, C \$20000 of the capital. Their net profit in one year is \$1680. Find the share of each in this profit.

\$8000 + \$12000 + \$20000 = \$40000, total capital.
\$1680 ÷ \$40000 = .042, rate of gain.
\$8000 × .042 = \$336, A's gain.
\$12000 × .042 = \$504, B's gain.
\$20000 × .042 = \$840, C's gain.

2. Three persons purchase a house for \$2800. A pays \$1200, B \$1000, and C \$600. The rent is \$224 a year. How much of this rent should each receive?

3. A, B, and C form a partnership, A furnishing $\frac{1}{3}$ of the capital, B $\frac{2}{5}$, and C the remainder. Find each partner's share of a gain of \$756 made in the business.

4. A firm loses \$4600 in a business transaction. The partners have the following investments in the firm: A \$10000, B \$3000, C \$6500, D \$3500. Find each partner's share of the loss.

5. Smith invested \$4200 in the firm of Smith and Jones, and his first year's profit was \$840,

while Jones's profit was $1120. How much capital did Jones invest?

6. In the firm of Smith, Brown, and Jones, they invest the following amounts: Smith $5000, Jones $4000, and Brown his labor and his experience, each partner sharing equally in the profits of the firm. Find each one's share of a profit of $1200 made the first year.

(Partnership is often considered under the topic "proportional parts" as the share of the profits or the losses is proportional to the amount invested. Further consideration of partnership will be given under this topic later in this book.)

EXCHANGE

276. Exchange is the system of making payments at different places without actually sending the cash.

Domestic exchange is exchange between cities of this country.

277. The forms of domestic exchange in common use are: checks, bank drafts, commercial drafts, and postal, express, and telegraphic money orders.

Examples of Domestic Exchange

278. If James Brown of Chicago wishes to pay me $500 in New York, he may do so by:

1. Sending me his check on a Chicago bank in which he has money on deposit. I can present

this check at a New York bank, and get the money. I may have to pay a small fee for collection of the check, unless the rate of exchange is in my favor.

2. He may go to any important bank in Chicago and buy a bank draft to my order on some particular bank in New York. This is a check drawn by one bank on another, called its correspondence bank. Mr. Brown may have to pay a little over $500 for the draft, unless the rate of exchange is in his favor at the time, and also a small fee to the bank for its trouble.

3. I may draw on Mr. Brown by means of a commercial draft, which would be written in the following form:

$500 NEW YORK, N.Y., June 5, 1909.

At sight pay to the order of the Importers and Traders National Bank of New York,

Five hundred~~~

they would return the draft to the New York bank, which would then notify me to "take up the draft" and charge the sum which they had advanced to my account. (Some banks will not cash the draft until it is collected.)

Commercial drafts are little used except in established lines of business, and where the reliability of the firms using them is well known.

279. Postal, express, and telegraphic money orders are used only for sending small amounts.

If Mr. Smith of Syracuse, N.Y., wishes to pay me $ 25, he can do so by money orders as follows:

1. Buy a postal money order at the post office in Syracuse and mail it to me. I could get this cashed at the post office in New York, after proper identification, or could deposit it in my bank for collection from the postal authorities. This postal money order would cost Mr. Smith $ 25.12, and as it can only be collected by me or by my order, it is a safe and a convenient way of sending small sums of money. This means of exchange is in general use where the sender has no regular bank account, and cannot, therefore, send his personal check.

The rates of exchange for postal money orders vary with the amount sent, the fee being 3¢ for $ 2.50 or less to 30¢ for $ 100.

No single money order is given for more than $ 100.

2. Mr. Smith could buy an express money order from an Express Company at the same cost, and mail it to me. I could then collect from the Express Company, either direct or through my bank.

3. If there was need for haste, Mr. Smith could transfer to me by telegraphic money order. This is expensive, costing high rates for the message and 1 % of the face of the draft for the making of the order. It is only used where there is immediate need for the transfer of the money.

280. Rates of exchange vary from time to time as the demand for drafts varies. If New York banks have large sums on deposit in Chicago, and wish to use it at home, they will sell drafts on Chicago for low rates. If they have little or no money in Chicago, and must send the cash by express to meet the draft, they will charge high rates of exchange. In ordinary times the rates of exchange vary but little, or from about 10¢ to $1 on a thousand dollars, either discount or premium.

281. Foreign exchange differs from domestic exchange chiefly as to currency, the manner of making the quotations, and the form of draft used. It is governed by the same general principles. The rates of foreign exchange change so often that the rate from day to day is published in the daily papers of all large cities.

282. Letters of credit are issued by international banking houses for the convenience of travelers.

A traveler leaving New York might deposit funds or other securities in a New York bank, and take out a letter of credit which he can draw against in foreign cities.

The identification is by means of his signature, which, written in the presence of the foreign bank officials, is compared with that written on the letter of credit.

His drafts, from time to time, are indorsed upon the letter of credit as they are paid by the bankers, so that each banker can tell how much remains to his credit at any time. Upon his return to New York he can take up the balance due him, or turn in his letter of credit to the bank and have the balance credited to his account.

283. Traveler's checks are also issued by express companies, and by a few tourist companies of established reputation. They are issued in fixed denominations of $10, $20, $50, etc., and at a fixed premium of $\frac{1}{2}\%$ of their face value.

These checks are more convenient for incidental expenses than the letter of credit, as they can be cashed at hotels, large stores, local places of exchange, etc., and need not be presented at a banking house during banking hours. They are secured against loss by a system of numbering,

and against use by any but the owner by the signature plan of identification. The value of these checks in foreign money is printed on their face, that of a $ 10 check being, —

£	s.	d.	Francs	Marks	Florins
2	0	6	51.25	41.25	24.15

This value, however, is subject to very slight changes, as the rates of exchange between the countries vary from day to day.

Exercises and Applications

284. 1. What must be paid in Memphis for a draft on Chicago for $ 5250, exchange being $\frac{1}{4}\%$ premium?

2. Find the cost of a bank draft for $ 300, when the exchange is at $\frac{1}{2}\%$ premium and the bank charges $\frac{1}{10}\%$ for its labor.

3. When exchange was at $\frac{3}{8}\%$ discount, I bought a bank draft for $ 50, the bank charging 15¢ for making out the draft. What did the draft cost me?

4. Find the cost of a New York draft for $ 800, exchange being $\frac{1}{8}\%$ premium and local bank charges being $\frac{1}{10}\%$.

5. Find the cost of a sight draft on Paris for 1500 francs, exchange being at 5.18. (This means 5.18 francs to the dollar.)

6. Find the cost of a demand draft on London for £ 50, exchange being 4.8525. ($ 4.8525 for £ 1.)

7. Find the cost of a 60-day draft on Berlin for 500 marks, exchange being $94\frac{1}{2}$. (4 marks for \$ .945.)

Note. Foreign exchange is quoted as follows:
England, — Cost of £ 1.
France, — Number of francs to the dollar.
Germany, — Cost of 4 marks.

8. A man in New York bought a bill of exchange for 1000 francs and sent it to his family in Geneva, exchange being $5.18\frac{1}{2}$. What did the bill cost him?

9. Find the cost of \$ 1000 in American Express checks, at $\frac{1}{2}$ % premium.

10. Find the cost of a sight draft on London for £25 10*s*., exchange at 4.86.

11. Mr. Smith buys farm implements listed for \$ 2500 with discounts of 30 % and 10 %, and pays by a draft at $\frac{1}{8}$ % premium. What does the draft cost him?

12. What will a draft for \$ 2750 cost, exchange at a premium of 40¢ per \$ 1000?

13. Smith and Jones draw on John Doe & Co. for \$ 750. The bank charges .1 % for collection. What are the net proceeds?

14. How large a demand draft on Paris will \$ 125 buy, exchange being 5.20?

15. A telegraphic money order costs twice the regular rate for a ten-word message plus 1 % pre-

mium on the order. What will it cost to send my friend in Syracuse $50 by a telegraphic money order from New York, the ten-word message rate being 25¢?

SUMMARY

The forms for keeping accounts vary according to the extent and character of the business.

In all forms, the title of the account is debit to what it receives and credit by what it gives.

The single-book and single-page form with adjacent columns for debit and credit entries of dollars and cents, and a common column for entry of items, is the simplest form, and is adapted to a very small business.

The two-book form, using a daybook and a ledger, is adapted to a slightly larger business.

Other books may be added for special purposes in large lines of business.

Double entry forms are used in large firms for more complete records.

A bill is a written statement of the items of an account, made in form to render to a debtor.

A statement is a written summary of an account, arranged in form to render to one interested.

A receipt is a written acknowledgment of something received.

Receipts and bills are often combined by receipting the bill when it is paid.

Printed blank forms are generally used for receipts, bills, statements, etc.

Banks are institutions established by law for the purpose of doing a certain line of business which is outlined in their charter.

The two main lines of banks may be classed as commercial or discount banks and savings banks.

Commercial banks receive money for safe keeping, pay out money of deposit upon order by checks from the depositors, discount notes, and perform many other business functions.

Savings banks receive money for investment and pay dividends or interest to their depositors.

Trust companies combine something of the functions of the commercial and the savings bank, and also act as trustees for estates, etc.

Bank discount is interest charged in advance on money loaned or advanced on commercial paper.

Bank discount is reckoned at a certain per cent per annum on the maturity value of the paper discounted.

The proceeds is the amount at maturity less the bank discount.

True discount is the difference between the maturity value and the present worth.

The present worth is the sum which, put at simple interest, will amount to the maturity value in the given time.

A corporation or company is an organization

formed by law for doing business. Its business is managed by directors or officers elected by its stockholders.

Certificates of stock showing the number of shares and the par value of each share are issued by corporations. These may be bought or sold. The frequent changes in the market value of these certificates lead to a great amount of speculation in the stock market for "active stocks."

Dividends are profits shared by the stockholders.

Par value means the face value of the stock.

Bonds are legally secured certificates payable at a fixed time, and bearing a fixed rate of interest upon their par value.

A broker is an agent who deals in stocks or bonds upon a commission.

Profit and loss is usually named in business in terms of per cent on the cost or the money invested.

Partnership is an association formed by two or more persons for doing business as a "firm" or a "house," and sharing the profits and losses in accord with the investments and terms of agreement.

Exchange is the system of making payments at a distance by means of commercial paper.

Domestic exchange is exchange in the same country.

The most common forms of domestic exchange are checks, bank drafts, commercial drafts, and postal, express, and telegraphic money orders.

Foreign exchange is exchange between different countries.

Foreign exchange differs from domestic exchange chiefly in the form of draft used, the terms of currency expressed, and in the manner of publishing the quotations showing the rates of exchange.

In foreign exchange, drafts are usually made in duplicate and sent by different mails, to provide against loss. When one is paid, the other is void.

Exchange on England is quoted by giving the cost in dollars of an English £.

Exchange on Germany is quoted by giving the number of cents that 4 marks will cost.

Exchange on France, Switzerland, Italy, etc., is quoted by giving the number of francs that can be bought for $1.

Exchange rate is usually quoted also at cable or sight rate, and on demand, 60, and 90 day rates.

CHAPTER XII

LONGITUDE AND TIME

285. The rotation of the earth causes the noon rays of sunlight to pass across its surface from east to west.

286. Our system of time is based upon the location of these noon rays, although the day begins and ends at midnight. Time from midnight to noon is reckoned as forenoon time, and from noon to midnight as afternoon time.

287. These noon rays pass entirely around the earth or over 360 degrees of longitude in 24 hours; thus passing across its surface at the rate of 15 degrees of longitude for each hour of time.

Upon this relation of longitude to time all calculations of longitude and time are based.

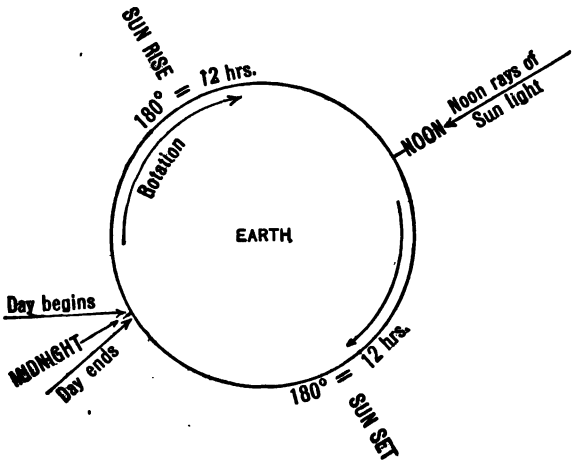
288. To determine the longitude of a ship at sea, the noon period is determined by observing the sun and noting the time when it rises to its highest point. An accurate timepiece, called a chronometer, is kept running at Greenwich time, upon the meridian of which our system of longitude is based.

The ship's officers, noting the difference in time shown by the chronometer and ascertained by observation of the sun, may compute the difference in longitude by calculating 15 degrees for each hour.

For example: At noon by the sun, as shown by the observation at sea, the chronometer shows that it is 1:30 by London or Greenwich time. This means that the noon rays passed London 1 hour 30 minutes before. As they pass at the rate of 15 degrees for an hour, they must have passed over $15° \times 1\frac{1}{2}$ or 22° 30′ of longitude. The longitude of the ship is, therefore, 22° 30′ west. The officers then calculate their latitude in a method too complex to be outlined here, and are thus able to locate their exact position on their charts, and to determine their course of sailing. Longitude may be found in other ways, but the one outlined is the one in common use when the sun is visible at noon.

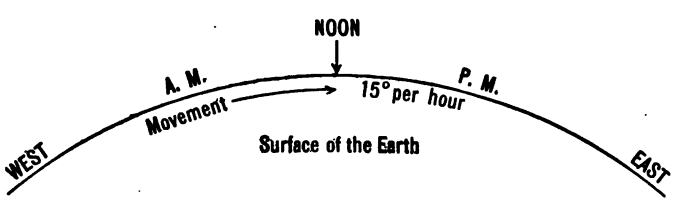
Standard time has come into general use for calculations on land, so that longitude and time is now but little used except for the purpose just outlined.

289. The following diagram shows the path of the noon rays around the earth on a parallel of latitude, and also shows our system of reckoning time based upon the noon point.



290. The following diagram shows a portion of the surface of the earth as passing by rotation under the noon rays of the sun, and indicates the direction in which the surface moves under these rays, as well as its rate of movement.

From this diagram exercises in longitude and time may be easily understood.



As the plane of the vertical rays of the noon period extends north and south at the time of the

equinoxes, and nearly so at all seasons of the year, it is sufficiently accurate for all places, except in the polar regions, to say that places having the same longitude have the same time, and that the more easterly place (not more than 180° distant) has the later time.

291. From the diagrams we may gather the following:

1. The rotation of the earth causes points on its surface to pass under the noon rays of the sun from the west.

2. The rate of movement is 15° per hour.

3. Our system of time is based upon the noon point.

4. The rotation of the earth causes the vertical plane of the noon rays to sweep across its surface in a northerly and southerly line, so that places in the same longitude have practically the same time.

292. For the change from longitude to time measure, or difference of time to longitude measure, the following table may be used:

Longitude Measure		Time Measure
360 deg.	=	24 hr.
15 deg.	=	1 hr.
1 deg.	=	4 min.
15 min.	=	1 min.
15 sec.	=	1 sec.

Exercises and Application of Longitude and Time

293. Change the following to time measure:

1. 4° 15′ 30″.
2. 120° 45′ 15″.
3. 18° 15′ 48″.
4. 72° 30′ 45″.
5. 2° 10′ 45″.
6. 25° 30′ 45″.
7. 45′ 18″.
8. 12° 25′ 40″.
9. 12° 35′ 20″.
10. 150° 45′ 30″.

294. Change the following to longitude measure:

1. 4 hr. 15 min. 12 sec.
2. 3 hr. 25 min.
3. 8 hr. 45 min. 30 sec.
4. 2 hr. 8 min 15 sec.
5. 6 hr. 15 min. 15 sec.
6. 2 hr. 45 min.
7. 1 hr. 15 min. 30 sec.
8. 7 hr. 40 min.
9. 5 hr. 45 min. 45 sec.
10. 9 hr. 30 min. 45 sec.

295. 1. What is the longitude of a ship whose chronometer, keeping Greenwich time, shows 1 : 45 at the time of the noon observations?

2. What is the longitude of the ship whose chronometer shows 8 : 12 P.M. at the time of the noon observation?

(*a*) If the ship is west from London and in latitude 38° N., what port is near?

3. At 11 : 12 a steamer sends a wireless message to another steamer, which is received by it at 11 : 00. The longitude of the first-named steamer is 35° 15′ W. What is the longitude of the second steamer?

4. The longitude of Nantucket lightship is 70° W. New York City has the time of the 75th meridian. When it is noon at the lightship, what is the time in New York?

5. A passenger steamer going to Glasgow from New York makes 8° 45′ of longitude from Wednesday noon to Thursday noon. How much will the passengers have to change the time of their watches? Will their watches be too slow or too fast on Thursday noon if they were correct on Wednesday noon?

6. A ship sailing from Naples to New York makes an average of 7° 15′ longitude each day. Will a passenger need to set his watch forward or backward each day, and how much?

7. A wireless message is sent from London at 10 : 05 A.M. to a ship which is in longitude 4° 30′ W. What is the sun time on the ship when the message is received?

8. A ship in 123° 45′ W. off San Francisco sends a wireless to that city at 11 : 50 A.M., sun time. What time will it be when the message is received in San Francisco, which has the standard time of the 120th meridian?

9. A wireless message is sent from Boston, having the standard time of the 75th meridian, to a ship which is in longitude 69° 15′ W. If the message is sent at 8 : 30 A.M., Boston time, what

is the sun time at which it is received on the ship?

10. Two ships, A and B, communicate by wireless, and find that their difference in time is 50 minutes, that of B being earlier. If A is in longitude 40° 15′ W., what is the longitude of B?

STANDARD TIME

296. As a matter of general convenience, most countries have adopted a standard time system by which the time of belts about 15 degrees wide, running north and south, is made uniform, the time of the standard meridian of the belt being used for the entire belt.

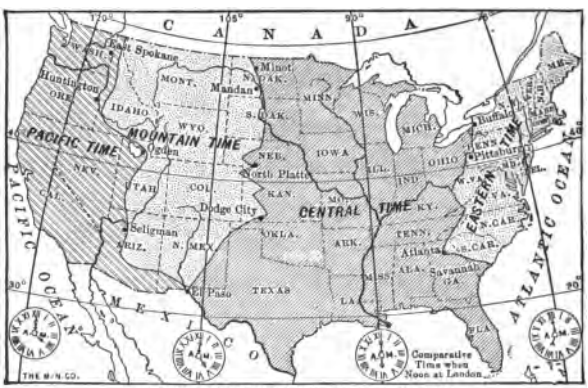
By this system the true time is disregarded for all points not on the standard meridians, and the standard time used for the entire belt. The time in one belt will differ from that in other belts by even hours, so that all difference of time in minutes and seconds is avoided.

297. The time in this country is based upon that of the 75th, 90th, 105th, and 120th meridians. About these standard meridians time belts are adjusted for convenience of railroad stations of importance, the change of time from one belt to another being made at important points.

298. Names have been given to these time belts as follows:

75th meridian standard, 5 hours slower than London, **Eastern.**
90th meridian standard, 6 hours slower than London, **Central.**
105th meridian standard, 7 hours slower than London, **Mountain.**
120th meridian standard, 8 hours slower than London, **Pacific.**

The following chart shows the time belts of this country.



STANDARD TIME IN THE UNITED STATES.

299. For railway time in Europe, Greenwich, or London time is used in Great Britain, Belgium, and Holland. Dublin time is used in Ireland. Paris time is used in France, this being $9\frac{1}{4}$ minutes faster than London time. Middle European time, one hour faster than London time, is used by the railroads of Sweden, Germany, Austria-Hungary, Servia, and Western Turkey. Eastern European time, two hours faster than London time, is used by the railroads in Eastern Turkey, Bulgaria, and Roumania. Japan uses the 135th E. longitude time, 9 hours faster than London time.

The Philippines use 120 E. longitude time, 8 hours faster than London, or 13 hours faster than New York time.

Australia has three standards, 120th E., 135th E., and 150th E. longitude, 8 hours, 9 hours, and 10 hours faster than London time.

Note. In some countries the railroad and the local or city time are not the same, as important cities use their own standards rather than the standards of the railroads.

Exercises and Applications of Standard Time

300. 1. When it is 10:00 A.M. at New York, what time is it at Chicago?

2. When it is 7 A.M. at San Francisco, what time is it at London?

3. When it is 5 P.M. at Cincinnati, what time is it at Boston?

4. When it is noon at Salt Lake City, what time is it in New York?

5. When it is 6 P.M. in London, what time is it in St. Paul?

6. When it is 8 P.M. in Berlin, what time is it in Chicago?

7. When it is 8:30 P.M. in Paris, what time is it in New York?

8. A man enters Buffalo at 9:10 Eastern time and leaves at 8:40 Central time. How much time did he spend in Buffalo?

9. A man enters Buffalo at 6 : 30 P.M. Central time and leaves at 8 : 15 Eastern time. How much time did he spend in Buffalo?

10. A man leaves Chicago at 7 : 10 A.M. Tuesday and reaches New York at 8 : 45 A.M. Wednesday. How many hours was he on the trip?

11. A man leaves New York at 8 : 45 A.M. Monday and reaches Chicago at 8 : 10 A.M. Tuesday. How long was he on the trip?

12. A steamship leaves New York at noon Saturday and reaches Glasgow the second Sunday following at 11 : 30 A.M. How long was the trip?

13. A steamship leaves Dover, England, at 10 : 15 P.M. Aug. 19, and arrives at New York at 8 : 30 A.M. Aug. 28. How long was the trip?

14. What time is it now (see clock for local time) in Chicago? London? Berlin? Paris? Brussels? Boston? St. Louis?

Note. The teacher may extend the exercises of the last question as far as desirable, permitting the pupils to use their geographies for the location of places and to consult the local time for their calculations.

15. Boston is in longitude 71° 03′ 50″ W., but uses the standard time of the 75th meridian. How much faster or slower is her standard time than the true time?

16. Washington is in longitude 77° 03′ 06′ W., but uses the standard time of the 75th meridian.

How much faster or slower is her standard time than the true time?

17. Manila is in longitude 120° 58′ 06″ E. New York is in longitude 74° 00′ 24″ W. What is their difference in time, using the exact time of each place? What is their difference in Standard Time?

Note. An irregular line corresponding generally to the 180th meridian is called the International Date Line. Ships passing this line going westward set their dates forward one day In crossing this line going east, they set their dates back one day. That is, when it is Sunday at the east of this line, it is Monday at the west of the line.

18. San Francisco has the Standard time of the 120th W. meridian, and Manila has the Standard Time of the 120th E. meridian. When it is 10 : 00 A.M. Tuesday in San Francisco, what is the date and time in Manila?

CHAPTER XIII

THE METRIC SYSTEM

301. The Metric System is a system of weights and measures expressed in a decimal scale. It is thus very convenient for computations, and is used by all important countries except the United States and Great Britain. It is the base for all international standards of comparison, and is used to a great extent in the arts and sciences. Its use in the United States is at present limited to the arts and sciences and to foreign trade correspondence and foreign accounts.

302. The principal unit is the **meter,** to which all other units are *related.*

The standard meter is carefully preserved in Paris, and accurate copies furnished to all civilized nations. The standards of our common measures are now based upon the meter (see denominate numbers in this book), which is, by our measure, 39.37 inches in length.

303. The principal units of the Metric System are:

The *meter* for length, 39.37 inches.

The *square meter* for surfaces, about 1.2 square yards.

The *cubic meter* for large volumes, about 1.3 cubic yards.

The *liter* for smaller volumes, about 1 quart.

The *gram* for weights, the kilogram or kilo, containing 1000 grams, being about 2.2 pounds avoirdupois.

The *are* (119.6 square yards) and the *hektare* (2½ acres) are used for land measure.

The *metric ton* (2205 pounds) is used for weighing hay, coal, etc.

304. In the tables of units, prefixes are used to show parts and multiples of the standard units.

Latin prefixes are used to show *parts*, as follows:

milli for .001.
centi for .01.
deci for .1.

Greek prefixes are used to show *multiples*, as follows:

deka for 10.
hekto for 100.
kilo for 1000.

(Myria is sometimes used for 10000.)

Tables are therefore made up by prefixing these terms to that of the standard unit, as follows:

milli- meter, or liter, or gram.
centi- meter, or liter, or gram.
deci- meter, or liter, or gram.
(standard unit.)
deka- meter, or liter, or gram.
hekto- meter, or liter, or gram.
kilo- meter, or liter, or gram.

305. The principal unit of length is the *meter* (39.37 inches).

TABLE OF LINEAR MEASURE

10 *millimeters* = 1 *centimeter* (about $\frac{3}{8}$ inches).
10 centimeters = 1 decimeter.
10 decimeters = 1 *meter* (39.37 inches).
10 meters = 1 dekameter.
10 dekameters = 1 hektometer.
10 hektometers = 1 *kilometer* (about .62 miles).

The units printed in italic *letters* are the ones in common use.

The millimeter is only used in very small measurements, to express thickness of wire, etc., as it is only about four hundredths of an inch in length.

For ordinary use we need to remember:

100 centimeters = 1 meter.
1000 meters = 1 kilometer.

306. In computations with metric measures which involve reduction from one unit to another in the same table, or change from one table to another, such as finding the number of liters in tanks, etc., of given dimensions, we make the reductions and changes very easily, as all is based upon the decimal scale.

The difficulties of our complex computations in denominate numbers would be almost entirely eliminated, if the metric system should come into general use in this country.

By the metric system, any measure or length may be expressed in terms of another by simply moving the decimal point. For example:

7245 meters = 7.245 kilometers.
7245 meters = 724500 centimeters.
7245 meters = 72.45 hektometers.
7245 meters = 724.5 dekameters.
7245 meters = 72450 decimeters.

Exercises and Applications of Units of Length

307. 1. Change 8435 meters to km.; to cm.; to dm.; to dkm.

2. Add 8 km., 4 m.; 16 km., 8 m.; 25 km., 75 m.; 15 km., 45 m.

3. From 3 m. subtract 15 cm.

4. Multiply 16 cm. by 125 and express the result in meters.

5. Divide 4 km. by 125 and express the result in meters.

6. Find the cost of 8.5 m. of cloth at $1.75 a meter.

7. A train runs at the rate of 75 km. an hour; what is its rate of speed per second?

8. A train runs 75 km. per hour; what is its rate of speed in miles per hour?

9. It is 626 km. from Geneva to Paris. This distance is passed by train in 10 hours. What is the average rate of speed in miles per hour?

10. Express the following in metric terms:

1. $2\frac{1}{2}$ miles.
2. 16 inches.
3. 4 yards.
4. 50 miles.
5. $\frac{1}{2}$ inch.
6. 50 yards.

7. 300 yards. 9. 8 inches. 11. 6 feet.
8. 500 miles. 10. 1 foot. 12. 100 feet.

SURFACE MEASURE

308. The principal unit of surface measure is the *square meter.*

309. The complete table may be made for surface measure by using 10×10 or 100 units for each next higher, and using the regular prefixes, as follows:

100 square millimeters = 1 square centimeter,
100 square centimeters = 1 square decimeter,
100 square decimeters = 1 square meter,
100 square meters = 1 square dekameter,
100 square dekameters = 1 square hektometer,
100 square hektometers = 1 square kilometer.

The only units that are in common use, and the table that will be most helpful in computations is as follows:

10,000 square centimeters = 1 square meter (about 1.2 sq. yd.).

1,000,000 square meters = 1 square kilometer (about .4 sq. mi.).

310. For land measure, the square dekameter or 100 square meters is used and called an *are.* This is about 120 square yards.

100 acres = 1 *hektare* or about $2\frac{1}{2}$ acres.

Exercises and Applications of Surface Measure

311. 1. At 20¢ a square meter, what will be the cost of painting a rectangular surface 24.5 m. long and 10.25 m. wide?

2. What is the area of a surface 14 cm. by 8 cm.?

3. Find the area of a triangular field whose base is 840 m. and its altitude 380 m.

4. A lot containing 84.5 acres is sold for 20¢ a square meter. Find its cost.

5. France has an area of about 530,000 square kilometers. Express this area in square miles.

6. Find the area in hektares of a triangular field whose base is 350 m. and its altitude 200 m.

7. Find the area in hektares of a rectangular field 200 m. long and 180 m. wide.

8. How many square meters in the surface of a sidewalk 1.75 meters wide and 40 meters long?

9. What is the area in ares of a rectangular field 50 m. long and 40 m. wide?

10. What will be the cost of one square kilometer of land at $300 a hektare?

11. What will it cost to cement the bottom of a cellar which is rectangular in form and has a length of 8 m. and a width of 5 m., the cement costing 18¢ a square meter?

12. How many square meters in a rectangular surface 20 feet by 15 feet?

13. How many square yards in a surface which is rectangular in form and is 20 m. long and 15 m. wide?

14. How many hectares in a field which contains 50 acres?

15. How many acres in a field which contains 100 hektares?

UNITS OF VOLUME

312. The principal unit of volume is the cubic meter (about 1.308 cubic yards).

313. The scale for units of volume is formed from that of linear measure, $10 \times 10 \times 10 = 1000$, each unit in cubic measure being 1000 times as large as the next smaller unit. The full table may be formed by using the regular prefixes and taking 1000 units for each next higher unit.

314. The following table shows the units in common use in the measure of volume by the metric system:

1000 cubic centimeters = 1 cubic decimeter.
1000 cubic decimeters = 1 cubic meter (about 1.308 cu. yd.).

In measuring wood the cubic meter is often called a stere.

Exercises and Applications

315. 1. Find the volume in cubic centimeters of a rectangular block of marble 25 cm. long, 20 cm. wide, and 8 cm. thick.

2. How many cubic meters of water will a tank hold which is 2 m. long, 3 dm. wide, and 25 cm. deep?

3. What will it cost to excavate a cellar 10 m. long, 9 m. wide, and 1.75 m. deep at 25¢ a cubic meter?

4. A tank is 3.5 m. long and 1.5 m. wide. How deep will the water need to be to contain 3 cubic meters of water?

5. How many steres of wood in a pile 8 m. long, 1.5 m. high, and 1 m. wide?

6. Find the cost of a piece of timber 4 m. long, 25 cm. wide, and 20 cm. thick at $20 per cubic meter.

7. How many cubic yards of earth will have to be removed to dig a cellar 12 m. long, 4 m. wide, and 2 m. deep?

8. How many cubic meters of earth will have to be removed to dig a cellar 15 feet long, 12 feet wide, and 6 feet deep?

UNITS OF CAPACITY

316. In measuring liquids, grain, etc., the principal unit of measure is the *liter*.

The liter occupies 1 cubic decimeter of space or volume.

The liter = 1.05 + liquid quarts, or .908 dry quarts of our measures.

317. TABLE OF MEASURES OF CAPACITY

10 milliliters = 1 centiliter
10 centiliters = 1 deciliter
10 deciliters = 1 *liter* (about 1 quart, U.S.)
10 liters = 1 dekaliter
10 dekaliters = 1 *hektoliter* (about 2.84 bushels, U.S.)

318. For the solution of problems and in general applications, the following are used:

1 cubic decimiter of space contains 1 liter.
100 liters = 1 hektoliter.

EXERCISES AND APPLICATIONS OF UNITS OF CAPACITY

319. 1. How many hektoliters of oats in a bin 3 m. long, 1.5 m. wide, and 2 m. deep?

2. How many liters of water in a tank 1.5 m. long, 75 cm. wide, and 20 cm. deep?

3. At 5¢ a liter, what will a hektoliter of vinegar cost?

4. Bought 24 hektoliters of corn at $1.50 per hektoliter and sold it at 60¢ a bushel. Find the gain or loss.

5. A merchant bought 400 liters of oil at 25¢ a liter and sold it at 35¢ a quart. Find his gain.

6. How many liters of beans will a box hold whose inside dimensions are 60 cm., 50 cm., and 40 cm.?

7. Bought 100 bushels of wheat at $1.05 per bushel and sold it at $3.25 per hektoliter. Find the gain.

8. A cylindrical tank has a height of 2 meters, and the area of its base is 5 square meters. How many hektoliters of water will it hold?

9. Find the cost of $1\frac{1}{2}$ hektoliters of wine at 90¢ a liter.

10. What will be the duty on 150 liters of wine at 60¢ a gallon?

UNITS OF WEIGHT

320. The principal unit of weight is the *gram*, which is the weight of 1 cubic centimeter of pure water at its greatest density.

321. A cubic decimeter of water is a *liter*, which weighs a kilogram or about $2\frac{1}{5}$ lb. avoirdupois.

322. While the entire table for units of weight may be formulated by using the regular prefixes, the units in common use and the table necessary for computations is:

1000 grams = 1 kilogram (about $2\frac{1}{5}$ lb. av.)
1000 kilograms = 1 metric ton (about 2205 lb.)
(Kilograms are often called kilos.)

Exercises and Applications of Units of Weight

323. 1. Express a gram as a decimal of an ounce.

2. Express a metric ton in tons U. S. measure.

3. Express 25 kilograms in U. S. pounds.

4. A boy weighs 115 pounds. About how much does he weigh in metric terms?

5. Find the weight of a cubic meter of water.

6. A metric ton of raw sugar costs $72. What is this cost per pound?

7. Find the weight of water in a tank which has inside dimensions of 2 m. length, 1.75 m. width, and contains a depth of 80 cm. of water.

Miscellaneous Exercises

324. 1. A barrel when empty weighs 10 kilos, and when filled with water it weighs 125 kilos. How many liters of water does it hold?

2. Find the weight of 15 liters of alcohol, which is .83 as heavy as water.

3. The distance from Rome to Pisa is 335 kilometers. Express this distance in miles.

4. The distance from New York to Buffalo is 410 miles. Express this distance in kilometers.

5. A barrel of flour weighs 196 pounds. Express this weight in proper metric terms.

6. How many steps, each $2\frac{1}{2}$ feet long, will a man take in walking a kilometer?

7. A 5-cent piece weighs about 5 grams. What is the weight in metric terms of $ 1 in 5-cent pieces?

8. Letter paper is about 8 by 10 inches. Express these dimensions in metric terms.

9. How many meters of ribbon will be required to make 64 badges, each containing 40 cm. of ribbon?

10. A wheel is 3.6 meters in circumference. How many times will it revolve in rolling a distance of 216 kilometers?

11. What must be the depth of a bin 1.5 m. wide, 5 m. long, to hold 7500 liters of grain?

12. How many boxes containing 2 grams each will be required to hold 1 kilo of quinine?

13. About how many pounds are there in 456 kilograms?

14. Bought 454 bu. of wheat at $1.50 a bushel and sold it at $4.25 a hektoliter. Find the amount of gain or loss.

15. What is the cost of a building lot 75 m. long and 62 m. wide at $25 an acre?

16. What price per pound is nearly equivalent to $2.50 per kilogram?

17. How many meters of lining that is 60 cm. wide will line 15 m. of silk that is 75 cm. wide?

18. How many hektares are there in a piece of land that is 150 meters long and 120 meters wide?

19. A liter of air weighs 1.292 grams. Find the weight of the air in a room 6.8 m. long, 5.2 m. wide, and 3 m. high.

20. Find the area in ares of a rectangular field that is 100 m. long and 50 m. wide.

21. At 18 cents a liter, what will be the cost of 25 gallons of olive oil?

22. Find the area of a circle which is 5 meters in diameter.

23. A gasoline engine burns about $\frac{1}{2}$ liter of oil per horse power per hour. How many gallons per horse power will it burn in 10 hours?

TABLE OF EQUIVALENTS

(For reference)

1 in. = 2.54 cm.	1 cm. = 0.3937 in.
1 ft. = 3.048 dm.	1 dm. = 0.328 ft.
1 yd. = 0.9144 m.	1 m. = 1.0936 yd. = 39.37 in.
1 mi. = 1.6093 km.	1 km. = 0.62137 mi.
1 sq. in. = 6.452 sq. cm.	1 sq. cm. = 0.156 sq. in.
1 sq. ft. = 9.2903 sq. dm.	1 sq. dm. = 0.1076 sq. ft.
1 sq. yd. = 0.8361 sq. m.	1 sq. m. = 1.196 sq. yd.
1 sq. rd. = 25.293 sq. m.	1 are = 3.954 sq. rd.
1 A. = 0.4047 hk.	1 hk. = 2.471 A.
1 sq. mi. = 2.59 sq. km.	1 sq. km. = 0.3861 sq. mi.
1 cu. in. = 16.387 cu. cm.	1 cu. cm. = 0.061 cu. in.
1 cu. ft. = 28.317 cu. dm.	1 cu. dm. = 0.0353 cu. ft.
1 cu. yd. = 0.7645 cu. m.	1 cu. m. = 1.308 cu. yd.
1 liquid qt. = 0.9463 l.	1 liter = 1.0567 liquid qt.
1 gal. = 0.3785 dk.	1 dkl. = 2.6417 gal.
1 dry qt. = 1.101 l.	1 l. = 0.908 dry qt.

1 bu. = 3.524 dkl.	1 hkl. = 2.8375 bu.
1 oz. av. = 28.35 g.	1 g. = 0.03527 oz. av.
1 lb. av. = 0.4536 kg.	1 kg. = 2.2046 lb. av.
1 gr. Troy = 0.648 g.	1 g. = 15.432 gr. Troy.
1 oz. Troy = 31.1035 g.	1 g. = 0.03215 oz. Troy.
1 lb. Troy = 0.3732 kg.	1 kg. = 2.679 lb. Troy.

The equivalents given above are those established by law for use in legal proceedings.

FOREIGN MONEY

325. Some knowledge of foreign money is necessary to the traveler and to one doing business with foreign countries.

In calculating duties, and in dealing with matters of either export or import, where comparison of prices in different countries is desirable, knowledge of the values expressed in the different systems of money is necessary.

326. The unit of English money is the pound or sovereign, which is equivalent to $4.86+ in our money. (This value varies from time to time as the rates of exchange vary, an English pound being worth from $4.85 to $4.87.)

TABLE OF ENGLISH MONEY

4 farthings (far.)	= 1 penny (*d.*)
12 pence	= 1 shilling (*s.*)
20 shillings	= 1 pound (£.)

The guinea, a coin struck in England in 1664 and issued until 1817, is still used for subscriptions, professional fees, etc. It was coined for 20 shillings, but passed for 21 up to 28 shillings.

Farthings are usually written as fractions of a penny.

The sign for pounds is written before the number, thus £3.

Shillings are often written before the slanting line, as 3/–, 3 shillings and 6 pence being written 3/6 and read 3 and 6 in common use.

327. For comparison, the following equivalents may be used:

Approximations	Close Equivalents
£1 = $5.	£1 = $4.8665.
1/– = 25¢.	1/– = $ .24333.
1*d.* = 2¢.	1*d.* = $ .0202+.

Exercises in English Money

328. 1. A Belfast merchant advertises damask table cloths, 2 yards square, at 2/9. What is the price in our money?

2. Lunch at 1/–, dinner at 1/6 is advertised. Express these prices in our money.

3. The fare from London to Dublin is quoted as follows: 1st class 53/6, 2d class 35/6, 3d class 29/6. What are these rates in our money?

4. How many articles costing 6*d.* each can be bought for £1 10*s.*? 34/6? 5/6? 7/–? 8/6? 53/6?

5. Bought a knife for 2/6 and gave the man a sovereign. How much change should I receive?

6. Paid 3/6 a pair for gloves. Bought 3 pairs, and gave the man £1. How much change should I receive?

7. Bought a raincoat in London for 25/6 and an umbrella for 3/4. What was the total cost? Express this cost in our money.

8. The cab fare in London is 1/– for a short drive. What is this value in our money?

9. Bought a suit of clothes in London for 75/6. What is this cost in our money?

10. Through tickets from Glasgow to Edinburgh, by Loch Lomond and the Trossachs, are quoted: 1st class 22/4, 2d class 17/6. What will this fare be in our money?

11. From London to Paris the fares are quoted as follows: By Dover and Calais, 56/8 and 39/8. By Folkestone and Boulogne, 50/– and 34/8. By Newhaven and Dieppe, 34/7 and 25/7 at night. Change these rates to U. S. money values.

329. The unit of French money is the *franc*, which is equivalent to $.193.

TABLE OF FRENCH MONEY

100 centimes (c.) = 1 franc (fr.) ($.193).

This system is used in Belgium, Switzerland, and Italy, except that the Italian coins, while having the same value, have different names.

FRANCE	ITALY
franc =	lira (plural lire).
centimes =	centesimi.

The values are written in the decimal form, as in our money, except that the sign is used after instead of before the amount, 4 francs 25 centimes being written 4.25 fr. 25 centimes may be written .25 fr. or 25 c., just as we write 25 cents, \$.25, or 25 ¢.

Exercises in French, Belgian, Swiss, and Italian Money

330. 1. From Geneva to Paris the fare is 1st class 70.10 fr., 2d class 47.35 fr. Express these rates in our money.

2. The charge for a telegram in Switzerland is 30 c. for the call and $2\frac{1}{2}$ c. for each word of the message. Find the cost of a ten-word message. Express this in our money.

3. The cab fare in Geneva for two to four persons is $2\frac{1}{2}$ fr. per hour. Find the cost of a cab for four persons from 10:00 to 12:00 o'clock.

4. From Rome to Naples is 162 miles by rail. The fare is 1st class 28.65 fr., 2d class 19.85 fr., 3d class 13.95 fr. Express these rates in our money. About how much in our money is the 2d-class rate per mile?

5. The rate for a gondola in Venice is 1.50 l. per hour. Find the cost of a ride in a gondola from 9:30 to noon.

6. Find the cost of $\frac{1}{2}$ liter of wine at 2.50 lire per liter. Express this cost in our money.

7. Bought a knife in Rome for 2.25 l. and gave the clerk a $5 gold piece. How much change should I receive in Italian money?

8. Bought three pieces of lace in Brussels, costing 45 fr., 8.75 fr., and 74.50 fr. Find the total cost. Express this cost in our money.

9. The fare by rail from Naples to Pompeii, a distance of 15 miles, is 2.75 fr., 1.90 fr., and 1.10 fr. Find the cost for four persons going second class. Express the cost in our money.

10. Bought a glass wine set in Venice for 75 fr. Paid the duty of 60 % at New York. What did the set cost me (including duty) in our money?

331. The unit of German money is the *mark*, equivalent to 23.8 cents in our money.

TABLE OF GERMAN MONEY

100 pfennigs (pf.) = 1 mark (m.) = $.238

The unit of Holland money is the *florin*, equivalent to about 40½ ¢ U. S. currency.

TABLE OF HOLLAND MONEY

100 cents (ct.) = 1 florin (Fl.) = $.405 +

332. The unit of the *new* Austrian currency is the crown (krone), equivalent to about 20 ¢ in our money. The krone is divided into 100 heller. The

value of the krone is about the same as that of the franc, and the value of the heller about the same as that of the centime (about $\frac{1}{5}$ ¢, U.S.).

Reckonings in Austria are still made, however, in the *old* currency terms, similar to that of Holland; the florin or gulden, of about 40 ¢ U. S. value, being divided into 100 kreutzers. The Austrian kreutzer has about the same value as the Holland cent, or about $\frac{2}{5}$ ¢ U.S.

Exercises in German, Holland, and Austrian Money

333. 1. Cab fare in Amsterdam is 80 C. per hour, and the tramway fare is $7\frac{1}{2}$ C. Express these rates in U.S. money.

2. The fare on the Rhine steamers between Cologne and Mayence, a distance of 188 kilometers, is 9 m. 50 pf. up and 12 m. down the river. Express the distance and the fare in terms of this country.

3. Bought 6 cigars at the Hague at 25 c. each, Dutch money. Gave the man 2 florins. How much change should I receive?

4. Lunch in Vienna cost me 2.50 crowns. What change should I receive from 2 florins?

5. Bought 3 bottles of cologne at Cologne for 7.50 m. What was the cost of each bottle in U.S. money?

6. Bought silver souvenirs in Holland costing

2.50 fl., 75 c.; 4.25 fl.; 8 fl.; 50 c.; and 3.25 fl. Find the total cost and the cost in U.S. money.

7. Bought a diamond ring in Amsterdam costing me 150 fl. What was the cost in U.S. money?

8. Bought a book of Rhine legends at Cologne for 3.50 m. What was the cost in our money?

9. Bought music in Vienna for 3.50 gulden. What was the cost in our money?

10. Bought a silver tea set at the Hague costing me 200 florins. Find the cost in our money.

COMPARISON OF THERMOMETERS

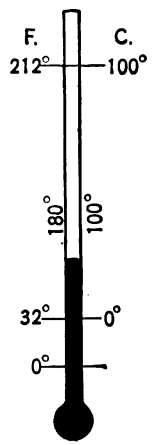
334. The thermometer in most common use in this country is that of the *Fahrenheit* scale, which is graduated at freezing point of water 32 degrees, and at the boiling point, 212 degrees.

By the Fahrenheit scale there are 180 degrees between the freezing and the boiling points of water, zero being 32 degrees below the freezing point.

335. The *centigrade* thermometer has a scale graduated at zero for the freezing point of water, and 100 degrees for the boiling point, thus making the temperature between freezing and boiling 100 degrees.

The centigrade scale is much easier to use, and is in common use in Europe, and in general use in this country for most scientific books and statements.

The following diagram shows the comparison of the two scales:



336. From the sketch it will be seen that by the scales:

FAHRENHEIT		CENTIGRADE
180°	=	100°
or 9°	=	5° (dividing each by 20),
1°	=	$\frac{5}{9}$° (dividing each by 9),
and $\frac{9}{5}$°	=	1°

To change from one reading to the other, however, we have to be careful and adjust the Fahrenheit readings to the reading of the *freezing point*, as that is the base of comparison.

For example: What centigrade reading corresponds with 77 degrees Fahrenheit?

77 deg. F. − 32 deg. = 45 deg., reading above freezing pt.
45 deg. F. × $\frac{5}{9}$ = 25°. The reading of centigrade scale.

What Fahrenheit reading corresponds with centigrade 20°?

20° ÷ $\frac{5}{9}$ = 36°, Fahrenheit degrees above freezing pt.
36° + 32° = 68°, Fahrenheit reading.

Exercise in Comparison of Thermometers

337. 1. The temperature of the class room should be about 70° Fahrenheit. What is this temperature by the centigrade scale?

2. The standard temperature of the human body is 98.6° Fahrenheit. Temperatures below 97 deg. or above 106 deg. indicate a bad state of health. Express these temperatures by centigrade scale.

3. Between December and March the temperature at Bermuda ranges from 16 to 19 deg. centigrade. What is this by Fahrenheit?

4. The mean temperature of San Francisco in December and January is about 55° Fahrenheit. What is this by centigrade?

5. By the Fahrenheit scale 60 deg. is called "temperate." What is this temperature by the centigrade scale?

6. The temperature in Paris is shown on a certain day by the centigrade thermometers to be 25 deg. What is this temperature by the Fahrenheit scale?

7. On July 4, 1909, the temperature in New York City was 98° Fahrenheit and in London 30° centigrade. What was the difference in temperature by the Fahrenheit scale?

8. When the temperature is −5° centigrade (5 deg. below zero), what is it by the Fahrenheit scale?

9. When the temperature is −9° Fahrenheit, what is it centigrade?

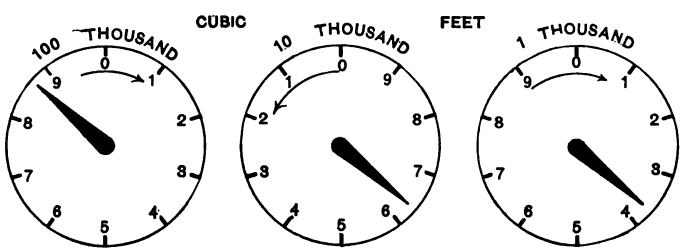
10. Change the following readings centigrade to Fahrenheit: 15 deg.; 30 deg.; 45 deg.; −10 deg.; 12 deg.; −20 deg.

READING METERS

338. Meters are now generally used for registering the amount of water, gas, electricity, etc., used, and it is desirable that one should be able to read them.

The general plan of the dials is the same for gas, water, and electric meters; the units which the dials register is always plainly marked on the face of the meter, as cubic feet, gallons, amperes, volts, kilowatts, etc.

The diagram shown below is that of a gas meter, but the reading would be on the same plan if it were for a different unit.



339. To read the meter, begin at the *left* dial, and read the figures on the dials in order, reading the figures that the indicators or hands *have passed.* This is the same plan that a railroad man uses in reading his watch. He notes the hour passed as shown by the hour hand, then notes the minutes passed as shown by the minute hand, then reads the number of seconds, and reads the time,

10 : 20 : 25, meaning 20 minutes and 25 seconds past 10 o'clock.

The meter reading shown in the diagram is 86300 cubic feet; or, if we estimate the distance shown on the right-hand meter that the indicator has passed beyond 3, we might read it 86375 cubic feet.

One common error in reading meters is caused by the words "1 thousand," "10 thousand," and "100 thousand" written above the dials on the meters. These show the amount registered by each dial for a *complete* revolution of the indicators. As each dial is divided into ten spaces, each number space is *one tenth* of the amount shown for a complete revolution of the indicator, or $\frac{1}{10}$ of the number shown above the dial.

If a clock should have printed above the dial 12 hours, it would be like the plan of the meter. This would indicate that the dial registered 12 hours, each division being $\frac{1}{12}$ of 12 hours or one hour.

Note. Drills in reading the meter may be had by study of the meter in every home in a city, or by making dials and turning the hands, or by placing sticks for hands on the dials shown above and changing the position of the sticks. An exercise from dials placed on the blackboard will be the best for class-room drill.

Exercises in computing cost from meter reading may be made, using the following prices:

Gas, 80¢ to $1.00 per 1000 cubic feet.
Water, 10¢ to 25¢ per 1000 gallons.

SUMMARY

Longitude and Time

The rotation of the earth causes the noon rays of sunlight to pass across the surface of the earth from east to west.

These noon rays pass around the earth or over 360 degrees of longitude in 24 hours.

The relation of longitude to time is, therefore, 360° to 24 hours or 15° to 1 hour.

Our system of reckoning time is based upon noon. The day begins and ends at midnight; time from midnight to noon being called forenoon time, and from noon until midnight being called afternoon time.

Places in the same longitude have nearly the same time at all seasons of the year. Difference in latitude has no relation to difference in time.

Longitude and time is used mainly as an aid to sailors in finding their positions at sea, and in locating places at sea.

Standard Time

As a matter of general convenience, most countries have fixed a system of standard time, by which the sun time of a standard meridian is used for a section on either side of it, the true sun time of places not on this meridian being disregarded.

As these standard meridians are 15° apart, the

differences in standards will always be in even hours, and the troublesome fractional differences of the old system of longitude and time will be avoided.

The standard time of most countries is based upon the time of the meridian of Greenwich, and is thus an even number of hours faster or slower than London time. The time in this country is based upon London time, as follows:

75th meridian, or eastern time, 5 hours slower than London.
90th meridian, or central time, 6 hours slower than London.
105th meridian, or mountain time, 7 hours slower than London.
120th meridian, or Pacific time, 8 hours slower than London.

The Metric System

The Metric System is a system of weights and measures in which the units are expressed on a decimal scale.

The system is thus very easy for computation, and is in general use in most countries except Great Britain and the United States.

In the United States the system is used for the table of money, has become the standard upon which our different units are compared, and is used in the arts and sciences to a great extent, as well as in foreign accounts, etc.

While it has been made the official system of the Philippine possessions, and its use in the entire country made legal, the attachment to the old system has as yet prevented its general use in the ordinary

affairs of trade and measurement in the United States.

The principal unit of the Metric System is the meter, to which all other units are related. This relation of units makes it easy to find equivalents of one table of measurement in terms of any other table.

The units of measurement in the metric system are:

The meter for length, 39.37 inches.
The square meter for surfaces, about 1.2 sq. yd.
The cubic meter for large volumes, about 1.3 cu. yd.
The liter for small volumes, about 1 quart.
The gram for small weights, about .03 ounces Av.
The kilogram for ordinary weights, about 2.2 pounds Av.
The are (119.6 sq. yd.) and the hektare (2.5 acres) are used for land measure.
The metric ton (2204+ lb.) for heavy weights.

In the table of units, the prefixes milli for .001; centi for .01; deci for .1; deka for 10; hekto for 100; kilo for 1000 are used, and the tables of units made by prefixing these terms to the name of the standard unit of the table.

Foreign Money

Except in Great Britain, foreign money tables are expressed in a decimal scale.

In Great Britain the pound ($4.86+) is divided into 20 units called shillings, the shillings being divided into 12 units called pence, and the pence

divided into 4 units called farthings. (The farthing is no longer coined, and is expressed as fractions of a penny.)

The unit of French money is the franc, which is divided into 100 units called centimes.

The franc is used in Belgium and Switzerland, and has general circulation in Italy, the Italian coin called the lira having the same value as the franc, or $.193 U.S. The Italian coin, the lira, is divided into 100 units called centesimi.

The unit of German money is the mark ($.238), which is divided into 100 units called pfennigs.

The unit of Holland money is the florin ($.405), which is divided into 100 units called cents.

Comparison of Thermometers

The thermometer in common use in this country has the Fahrenheit scale of graduation, the freezing point of water being 32°, the boiling point 212°, and 0° on this scale 32° below the freezing point of water.

The thermometer in common use in Europe and in use in the arts and sciences in this country has the centigrade scale of graduation. By this scale the freezing point of water is 0° and the boiling point 100°.

For comparison of the readings of these thermometers, we use the freezing point of water as a base, and the equation 9° Fahrenheit = 5° centi-

grade, using care to adjust the Fahrenheit readings to the freezing point as a base.

Reading Meters

In reading gas and water meters, we begin at the left-hand dial and read the dials in order, reading the figure which the index hand of the dial has *passed*. The heading on the meter over each dial gives the amount registered by that dial for a complete rotation of the index hand, each division of the dial or each figure on its face standing for *one tenth* of the amount printed over the face of the dial.

CHAPTER XIV

INTRODUCTION OF THE USE OF ALGEBRA AS AN AID TO ARITHMETIC

340. Some of the elementary principles of algebra may be easily learned that will be helpful in the solution of problems in arithmetic and valuable in the practical applications of life. Among these are:

1. Easy algebraic conventions.

2. Use of the equation in the solution of problems, where there are one, two, or three unknown quantities.

341. Algebra is universal arithmetic, dealing with the general rather than being limited to the special application of a particular problem or exercise.

By algebra, when one problem or exercise is solved, a form is established by which all problems or exercises of a similar type may be solved.

342. In the solution of problems by algebra, there are two main steps, viz.:

1. Statement of the terms of the problem in equation form.

2. Solution of these equations to determine the value of the unknown terms in the simplest form.

343. The algebraic conventions desirable for introductory use are:

1. Unknown quantities are represented by the last letters of the alphabet, usually x, y, and z.

2. Multiplication is usually represented without the use of the sign, by writing the terms together. $3x$ means 3 times x, just as 3 lb. means 3 times 1 lb.

3. Division is usually represented in the fractional form, $x \div 3$ being written $\frac{x}{3}$, etc.

4. Addition and subtraction are represented as in arithmetic, except that the sign of subtraction is used in a more extended manner, and operations not actually possible by arithmetic are indicated as to result by the use of the minus sign. For example, while we cannot subtract 5 from 3, we often in algebra indicate the result as -2.

344. In simplifying the terms of an equation we may:

1. Add the same quantity to both members.

2. Subtract the same quantity from both members.

3. Multiply both members by the same quantity.

4. Divide both members by the same quantity.

These facts are shown by their application to any simple equation, such as,

$$12 = 12.$$

Add 3 to each member, it becomes $15 = 15$.
Subtract 4 from each member, it becomes $8 = 8$.
Multiply both members by 2, it becomes $24 = 24$.
Divide both members by 6, it becomes $4 = 4$.

345. Transposition is the operation of changing a quantity from one member of the equation to the other. This may be done by *changing its sign* (from plus to minus or minus to plus).

Examples

(*a*) $x - 5 = 3$

$5 = 5$

(*b*) $x = 3 + 5$

(Add 5 to each member. -5 plus $5 = 0$.)

The 5 of equation (*a*) has been transposed and appears in equation (*b*) on the opposite side, *with its sign changed.*

(*a*) $3x = 2x + 5$

$2x = 2x$

(*b*) $3x - 2x = 5$

(subtracting $2x$ from both members).

Equation (*a*) has the $2x$ transposed in equation (*b*) *with its sign changed.*

When a quantity is changed from one member of the equation to the other for convenience in the solution of the equation, we must *change the sign*

of the quantity *when we cross the "bridge"* or sign of equality.

346. It is often desirable to clear equations of fractions. This is done by *multiplying each quantity* by such a number as will leave no fractional results. The number necessary for use as a multiplier to clear of fractions is the least common multiple of all of the denominators of the terms used in the equation.

Examples

(*a*) $\frac{x}{4}+\frac{3}{8}=\frac{7}{8}$.

(*b*) $2x+3=7$ (multiplying by 8).

(*c*) $2x=7-3$ (transposing the "3").

(*d*) $2x=4$ (uniting terms).

(*e*) $x=2$ (dividing by 2).

347. Find the value of x in the following:

1. $5x+22-2x=31$.
2. $10x-6x+14=62$.
3. $4x+20-6=34$.
4. $x-1=4x-91$.
5. $\frac{x}{2}-3+\frac{x}{3}=2$.
6. $\frac{x}{4}+\frac{x}{8}-\frac{x}{6}=\frac{5}{12}$.
7. $\frac{5x}{8}+\frac{1}{4}=\frac{11}{6}+\frac{7x}{12}$.
8. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=130$.
9. $\frac{2x}{3}=14$.
10. $\frac{6x}{7}=48$.

11. $5x+16-6x=16+24-6x.$

12. $11x-60+5x+17-2x-3x+41 = 106+2x.$

13. $\frac{3x}{4}-\frac{x}{9}=23.$

14. $\frac{4x}{5}+12=\frac{3x}{8}+29.$

15. $\frac{x}{2}+\frac{x}{3}+\frac{3x}{4}+\frac{4x}{5}=143.$

16. $3\ (x+1)+4\ (x+2)=6\ (x+3).$

Note. First clear of parenthesis by performing the multiplications indicated, then simplify the resulting equation.

17. $5\ (x+1)+6\ (x+2)=6\ (x+7).$

348. Solve the following problems by arithmetic and by algebra:

Note. In the introductory applications of algebra, it has been found to be valuable to compare the solution of problems by arithmetic with the solution of the same problems by algebra. This exercise is required by the Course of Study of the city of New York, and may well have consideration in all schools. Each pupil may be required to perform both solutions of all problems, or the class may be divided into sections, one section solving certain problems by one method while another section uses the other method.

1. The sum of two numbers is 84, and their difference is 14. Find the numbers.

SOLUTION BY ALGEBRA

Let $x=$ less number.

Then $x+14=$ greater number,

and (*a*) $2x+14=84.$

(*b*) $2x = 84 - 14.$

(*c*) $2x = 70.$

(*d*) $x = 35$, the lesser number.

(*e*) $x + 14 = 49$, the greater number.

Proof: $35 + 49 = 84.$

$49 - 35 = 14.$

(Compare this solution with that of arithmetic, which it is not necessary to give here.)

2. A man having 10 acres of land sold each of several men $\frac{3}{4}$ of an acre and had $2\frac{1}{2}$ acres left. To how many did he sell?

SUGGESTION. Let x = the number to whom he sold.

Then, $\frac{3x}{4} + 2\frac{1}{2} = 10.$

3. The perimeter of a field is 200 rods, and three times the width is twice the length. Find the dimensions of the field.

SUGGESTION. Let x = the length of the field.

Then $\frac{2x}{3}$ = the width,

and $2\left(x + \frac{2x}{3}\right) = 200$ rods.

4. A man paid \$5 for two hats. One cost $\frac{2}{3}$ as much as the other. Find the cost of each hat.

5. My salary is \$900 this year, which is 25% more than it was last year. What was my salary last year?

SUGGESTION. Let x = salary of last year.

Then $$x + \frac{25\,x}{100} = \$\,900.$$

5. A man spends $\frac{1}{6}$ of his income for rent, $\frac{1}{5}$ for groceries, etc., and has \$ 1140 left. What is his income?

7. James is 16 years younger than his brother Joseph, but 3 times the age of Joseph is equal to 5 times that of James. Find the age of each.

8. A quantity of dress goods was sold for \$ 67 or at a loss of 15 %. What did it cost?

9. Find the principal that will earn \$ 450 interest in 3 yr. at 6 %.

SUGGESTION. Let x = the principal.

Then $$\frac{18\,x}{100} = \$\,450.$$

10. What principal will produce \$ 15 interest in 2 yr. 6 mo. at 6 %?

11. A has \$ 100 and B has \$ 20. How much must A give to B in order that each may have the same sum?

SUGGESTION. Let x = the sum given to B.

Then $$\$\,100 - x = \$\,20 + x.$$

12. A man wishes to divide a steel bar 12 feet long into two parts so that the longer shall be 8 inches longer than the other. How long should he make the shorter part?

13. A can do a piece of work in 2 days, and B can do it in 3 days. How long will it take both together to do the work?

SUGGESTION.

Let x = the number of days for both.

Then $\frac{1}{x}$ = the *part* both together do in one day,

and $$\frac{1}{x}=\frac{1}{2}+\frac{1}{3}.$$

14. A can do a piece of work in 5 days, B in 4 days, and C in 3 days. How long will it take them together to do the work?

15. Divide \$ 2000 between two persons so that one shall have $\frac{7}{9}$ as much as the other.

16. A steamer sails a mile downstream in 5 minutes, and a mile upstream in 7 minutes. How far downstream can she go and return in one hour?

17. In a mixture of $62\frac{1}{2}$ gallons, composed of milk and water, the water is 25 % of the milk. How many gallons are there of each?

18. How shall I mark goods which cost me \$ 240 that I may fall 10 % from the marked price and still make 20 % profit?

SUGGESTION. Let x = the marked price.

Then $$x-\frac{10\,x}{100}=\$\,240+\left(\frac{20}{100}\text{ of }\$\,240\right).$$

19. How shall I mark hats which cost me \$ 2 so that I may fall 20 % from the marked price and still make 20 % profit?

20. Divide 15 apples between A and B so that $\frac{1}{2}$ of A's number will equal $\frac{1}{3}$ of B's.

SUGGESTION. Let x = A's number.

Then $15 - x$ = B's number,

and $$\frac{x}{2} = \frac{15 - x}{3}.$$

21. The width of a room is $\frac{3}{4}$ of its length, and the distance around the room is 70 feet. Find the length and width.

22. A dealer sold a lot of rugs for \$ 9360, which was $\frac{1}{3}$ more than they cost him. How much did they cost him?

TWO UNKNOWN QUANTITIES

349. Sometimes it is convenient to use more than one unknown quantity in making the statement and forming the equations for the solution of problems. Two or more unknown quantities may be used in the statements, provided that we can form from the terms of the problem *as many independent equations* as there are *unknown quantities* in the statement.

350. Often the same example may be worked by either one or two unknown quantities. For comparison of the processes, both are shown for the following problem:

Divide 48 into two such parts that if the less be divided by 4 and the greater by 6, the sum of the quotients will be 9.

By One Unknown Quantity

Let x = the less.

Then $48 - x$ = the greater,

and (*a*) $\frac{x}{4} + \frac{48 - x}{6} = 9.$

(Clearing of fractions) (*b*) $3x + 96 - 2x = 108.$

(Transposing) (*c*) $3x - 2x = 108 - 96.$

(Uniting) (*d*) $x = 12$, the less.

Then $48 - x = 36$, the greater part.

By Two Unknown Quantities

Let x = the less

and y = the greater part.

Then (*a*) $x + y = 48,$

and (*b*) $\frac{x}{4} + \frac{y}{6} = 9.$

(Clearing (*b*) of fractions) (*c*) $3x + 2y = 108$

(Multiplying (*a*) by 2 to enable us to eliminate the y by subtraction) (*d*) $2x + 2y = 96$

The less (subtracting (*d*) from (*c*)) (*e*) $x = 12,$

(Substituting value of x for x in (*a*)) (*f*) $12 + y = 48$

(*g*) $y = 48 - 12$ (transposing)

(*h*) $y = 36$, the greater (uniting)

$x = 12$; $y = 36$. *Ans.*

351. In the solution of problems where we have two unknown quantities, we must

1. Form at least *two* independent equations from the statements of the problems.

2. So combine these equations that we may eliminate one of the unknown quantities.

3. After finding the value of one unknown quantity, we substitute this value for the quantity in one of the equations of the statement, and thus find the value of the other unknown quantity.

352. Elimination may be made by several methods, but only one need be outlined here, viz: Elimination by addition and subtraction. By this method we

1. Multiply or divide the equations by such numbers as to cause each equation to contain the same number of one of the unknown quantities.

2. Then by adding or subtracting the equations thus formed we eliminate this unknown quantity.

Note. If the signs of the quantity to be eliminated are *alike*, we *subtract* the equations to eliminate the quantity.

If the signs of the quantity are one plus and the other minus, we *add* the equations to eliminate the quantity.

Examples of Combinations necessary for Elimination

1. (*a*) $3x-y=7$

   (*b*) $x-y=1$ (Subtract to eliminate y.)

   $2x=6$

   $x=3$

2. (*a*) $3x-y=7$
(*b*) $x+y=5$ (Add to eliminate y.)
$4x=12$
$x=3$

Exercises with Equations of Two Unknown Quantities

353. Find the value of x and y:

1. (*a*) $2x+4y=28$.
(*b*) $4x+3y=31$.

2. (*a*) $3x+y=41$.
(*b*) $7x-y=79$.

3. (*a*) $2x+2y=26$.
(*b*) $7x-y=67$.

4. (*a*) $5x-2y=8$.
(*b*) $7x-y=13$.

5. (*a*) $9x+2y=20$.
(*b*) $3x+y=7$.

6. (*a*) $7x+4y=40$.
(*b*) $3x+2y=18$.

7. (*a*) $11x+8y=37$.
(*b*) $5x+6y=18$.

8. (*a*) $5x+2y=49$.
(*b*) $3x-2y=23$.

9. (*a*) $y+2x=18$.
(*b*) $y-2x=2$.

10. (*a*) $7x+4y=58$.
(*b*) $9x-4y=38$.

11. (*a*) $4x+3y=22$.
(*b*) $-4x+2y=-12$.

12. (*a*) $6x+5y=128$.
(*b*) $3x+4y=88$.

13. (*a*) $4x+6y=46$.
(*b*) $5x-2y=10$.

14. (*a*) $5y+3x=93$.
(*b*) $3y+4x=80$.

15. (*a*) $x+\frac{y}{2}=8$.
(*b*) $\frac{x}{2}+y=7$.

16. (*a*) $4y+x=102$.
(*b*) $y+4x=48$.

354. In the following exercises, first clear the equations of fractions and then eliminate by addition and subtraction.

1. (*a*) $\frac{x}{4}+\frac{5y}{6}=14.$ (*b*) $\frac{x}{8}+\frac{y}{6}=4.$

2. (*a*) $\frac{3x}{7}+\frac{2y}{3}=5.$ (*b*) $x-y=4.$

3. (*a*) $x+\frac{y}{2}=8.$ (*b*) $\frac{x}{2}+y=7.$

4. (*a*) $\frac{x}{4}+\frac{y}{5}=30.$ (*b*) $\frac{x}{8}+\frac{y}{15}=13.$

5. (*a*) $\frac{4x+3y}{9}+y=4.$ (*b*) $x+\frac{8x-2y}{5}=7.$

6. (*a*) $x+\frac{x}{2}-\frac{y}{5}=10.$ (*b*) $\frac{x}{8}+\frac{y}{10}=2.$

7. (*a*) $\frac{y}{7}-\frac{x}{3}+5=2.$ (*b*) $x+\frac{y}{5}=17\frac{4}{5}.$

8. (*a*) $\frac{y}{2}+\frac{x}{3}=6\frac{1}{6}.$ (*b*) $\frac{y}{4}-\frac{x}{7}=\frac{1}{2}.$

Problems with Two Unknown Quantities

355. 1. A man paid \$ 44 for 6 barrels of flour and 4 bushels of oats; and \$ 32 for 3 barrels of flour and 7 bushels of oats. What were the prices of each?

SUGGESTION. Let $x=$ the price of the flour,
and $y=$ the price of the oats.

Then (*a*) $6x+4y=\$\,44.$
(*b*) $3x+7y=\$\,32.$

2. What fraction is that, to the numerator of which, if 4 be added, the value is $\frac{1}{2}$; but if 7 be added to its denominator, its value is $\frac{1}{5}$?

SUGGESTION

Let x = the numerator of the fraction,
and y = the denominator.
$\frac{x}{y}$ = the fraction.

Then (*a*) $\frac{x+4}{y}=\frac{1}{2}$.

(*b*) $\frac{x}{y+7}=\frac{1}{5}$.

3. What numbers are those whose difference is 7 and sum 33?

4. Divide 75 into two such parts that three times the greater will exceed seven times the less by 15.

5. If 7 bushels of wheat and 10 bushels of rye cost \$15; and 4 bushels of wheat and 5 bushels of rye cost \$8, what are the prices of each?

6. Half the sum of two numbers is 20, and three times their difference is 18. Find the numbers.

7. 1000 men and women are employed in a factory. The average daily wage of the men is \$2.50 and of the women \$1.50. If the wages amount to \$2340 per day, how many men and how many women are employed?

8. At a certain election 375 persons voted for two candidates, and the candidate elected had a majority of 91. How many voted for each?

9. An English wine merchant finds that if he mixes sherry and brandy in proportion of 2 to 1, the mixture will be worth 78*s.* per dozen; but if he mixes them in proportion of 7 to 2, he can get 79*s.* a dozen. What is the price of each liquor per dozen?

SUGGESTION. Let x = the price of sherry.
y = the price of brandy.

Then $$\frac{2x+y}{3}=78.$$

$$\frac{7x+2y}{9}=79.$$

(The number of gallons in each combination has to be considered in forming the equations.)

10. A and B have together \$100. If A were to spend $\frac{1}{2}$ of his money, and B $\frac{1}{3}$ of his, they would then have only \$55 between them. How much money has each?

RATIO AND PROPORTION

356. The importance of Ratio and Proportion in mathematics, and in the logical consideration of any of the problems of life, is not easily overestimated.

The relation or ratio of a cause to an effect in

any line being understood, we may calculate either generally or definitely the extent of a similar cause to produce a given effect, or the extent of an effect that will be produced from a given cause.

To our grandfathers "The Rule of Three," known to us as simple proportion, was the key to mathematics and the base for logical reasoning. We may well give it very careful consideration. In multiplication and division, in the true consideration of the fractional idea, and in the solution of nearly all problems, we are dealing with ideas of proportion.

357. Ratio is the relation of one quantity to another of the same kind.

Ratio is expressed by the *quotient* of the first quantity compared divided by the second.

Ratio may be written by use of the colon or by the sign of division or in fractional form, the ratio of 3 to 4 being written:

$$3 : 4, \text{ or } 3 \div 4, \text{ or } \tfrac{3}{4}.$$

Proportion is an Equality of Ratios

358. In fractions we have learned that $\frac{4}{6} = \frac{2}{3}$, which is really a proportion, and means that the relation of 6 parts to 4 parts is the same as the relation of 3 parts to 2 parts.

This may be expressed in the form of a proportion as follows:

(*a*) 4 : 6 as 2 : 3, or 4 : 6 = 2 : 3 (read the ratio of 4 to 6 = the ratio of 2 to 3).

(*b*) 6 : 4 as 3 : 2, or 6 : 4 = 3 : 2.

In a problem such as: If 3 oranges cost 12¢, what will 6 oranges cost? we may think of the *ratio* of the number of oranges in each case, and thus determine the ratio of the cost.

The child, when he reasons "twice as many oranges, therefore twice as much cost," has used both ratio and proportion.

Written in form of proportion, the statement is in the following form:

3 oranges : 6 oranges as 12¢ : how many cents?

or

3 oranges : 6 oranges = 12¢ : x

Three of the quantities are given to find the fourth, hence the old name to the form of solution, "The rule of three."

359. To find the required or unknown term in a proportion, we may arrange the ratios in fractional form, clear of fractions, and solve as in algebra. In the proportion, $5 : 7 = 15 : x$,

Expressed in fractional form, it is $\frac{5}{7} = \frac{15}{x}$.

Clearing of fractions, it is $5x = 105$.

Dividing both terms by 5, it is $x = 21$.

If we note the result, after we clear of fractions, we will see that it expresses the product of the ex-

tremes or "outside" terms equal to the product of the means or "interior" terms.

From this observation we fix the principle:

The product of the extremes = the product of the means.

In the solution of statements in the form of a proportion, we may apply the principle without stating in fractional form and clearing of fractions each time, and thus form the equation direct.

Exercises and Applications of Ratio

360. 1. What is the ratio of 5 to 12? *Ans.* $\frac{5}{12}$.

2. What is the ratio of $\frac{1}{2}$ to $\frac{1}{8}$?

3. What is the ratio of 1 inch to 1 foot (12 inches)?

4. What is the ratio of 1 quart to 1 bushel?

5. What is the ratio of 3 lb. to 4 oz. av.?

6. What is the ratio of 3 lb. to 4 oz. Troy?

7. What is the ratio of 1 yard to 1 meter?

8. What is the ratio of 1 meter to 1 yard?

9. What is the ratio of the diameter to the circumference of a circle? Of the circumference to the diameter?

10. What is the ratio of a kilometer to a mile? A mile to a kilometer?

11. What is the ratio of 5 pounds of sugar to a kilogram of sugar?

12. What is the ratio of 3 quarts of wine to 3 liters of wine?

13. What is the ratio of 6 months' interest to 9 months' interest on the same sum at the same rate?

14. What is the ratio of the cost of 375 lb. of coal to the cost of 1 ton of coal?

15. What is the ratio of the amount of coal that can be bought for $15 to that which can be bought for $10?

16. What ratio has the *area* of a field 4 yd. square to the *area* of a field 8 yd. square? To the area of one 12 yd. square?

17. What ratio has the *area* of a circle 2 ft. in diameter to the area of a circle 4 ft. in diameter? To the area of one 6 ft. in diameter? To the area of one 8 ft. in diameter?

18. What ratio has the *volume* of a 4-inch cube to the volume of a 2-inch cube? To the volume of a 6-inch cube?

19. What ratio has the area of a field 4 rods square to 4 square rods?

20. What ratio has $\frac{1}{2}$ mile to 40 rods?

21. When 25% profit is made, what ratio has the cost to the gain? What ratio has the gain to the cost?

22. When a profit of 25% is made, what ratio has the selling price to the cost? What ratio has the cost to the selling price?

23. What ratio has the cost of $\frac{3}{8}$ of a farm to the cost of the whole farm? What ratio has the cost of the whole farm to the cost of $\frac{3}{8}$ of the farm?

24. At 6% interest, what ratio has the interest to the principal?

25. At a discount of 20%, what ratio has the cost to the list price? The list price to the cost?

Exercise and Applications of Proportion

361. Find the value of the missing term:

1. $16 : 3 = 48 : x$.
2. 1 lb. : $\frac{1}{8}$ lb. = $12¢ : x$.
3. 1 mile : 80 rd. ($\frac{1}{4}$ mi.) = 2 min. : x.
4. 3 bu. : 3 pk. = \$1.50 : x.
5. 4 yd. : 4 meters = \$2 : x.
6. 380 lb. : 1 ton = x : \$7.50.
7. \$7.50 : \$5 = 1 ton : x.
8. $\frac{1}{2} : .375 =$ \$1.50 : x.
9. $\frac{5}{8} : .125 = 75¢ : x$.
10. $\frac{7}{8} : \frac{3}{16} = 50¢ : x$.
11. 4 liq. qt. : 3 liters = $80¢ : x$.
12. 3 acres : 2 hektares = \$ 225 : x.
13. 5 lb. coffee : 1 kilogram coffee = \$ 1.50 : x.
14. Int. on \$ 500 at 6 % for 3 mo. : int. on \$ 500 at 4 % for 3 mo. = \$ 7.50 : x.
15. Area of field 4 rd. square : area of field 2 rd. square = \$ 80 : x.

16. Cost of $\frac{3}{8}$ of farm : cost of farm = \$ 450 : x.

17. Cost of whole farm : cost of $\frac{3}{8}$ of farm = \$ 7200 : x.

18. Area of a 3-in. square : area of a 4-in. square = 9 : x.

19. $\frac{1}{2}$ of cost : 75 % of cost = \$ 50 : x.

20. 80 % of value : $\frac{2}{5}$ of value = \$ 240 : x.

Suggestions for making Proportional Statements from the General Terms of a Problem

362. 1. Make the third term like the answer sought.

2. If by the conditions of the problem this answer will be *less* than the third term, place the *greater* number of the other ratio as the *first* term; if not, place the greater number as the second term.

Suggestions for making Computations from Proportional Statements, which have been made as suggested above

363. 1. Make the computations with the numbers considered as abstract and then name the terms of the result from the problem or from the third term of the statement.

2. Cancellation of equal factors from the *first* and either the second or third term may be made to shorten the computations. *Do not* cancel from the second to the third term.

Applications and Problems

364. 1. If 24 lb. of sugar cost \$ 3.60, how much must be paid for 5 lb. of sugar at the same rate?

SUGGESTED STEPS FOR STATEMENT

——— : ——— = \$ 3.60 : x. *Ans.* (less).

Therefore, 24 lb. : 5 lb. = \$ 3.60 : x.

2. If 95¢ will buy one bushel of wheat, how many bushels will \$ 12.35 buy at the same rate?

(*a*) ——— : ——— = 1 bu. : x. bu. *Ans.* (more).

(*b*) 95¢ : 1235¢ = 1 bu. : x. bu.

3. How long will it take 24 men to do a job of work that 8 men can do in 12 days?

——— : ——— = 12 days : x days. *Ans.* (less).

24 men : 8 men = 12 days : x days.

4. If $\frac{3}{8}$ of a farm cost \$ 720, what will $\frac{7}{8}$ of the farm cost at the same rate?

5. If a train runs $96\frac{3}{4}$ miles in $3\frac{1}{2}$ hours, how far will it run in $5\frac{7}{8}$ hours at the same rate of speed?

6. If the distance of 48 miles is represented on a map by $1\frac{3}{4}$ inches, what distance will be represented on the same map by $7\frac{7}{8}$ inches?

7. If the net profit from spraying 6 acres of potatoes is \$ 138, what will be the profit from spraying 15 acres?

8. If $\frac{3}{8}$ of an acre of land cost \$ 30, what will 6 acres cost at the same rate?

9. If a loaf of bread weighs 8 ounces, when flour is $4.50 a barrel, how much should it weigh when flour is $6 a barrel?

10. A farmer finds that he uses 3 pk. of seed on $\frac{1}{4}$ acre of ground. At this rate how much seed will he need for $3\frac{3}{4}$ acres?

11. If $\frac{3}{8}$ of an acre of land cost $60, what will $45\frac{3}{4}$ acres cost at the same rate?

12. If 2 yd. of cloth cost $1.50, what will 2 meters of cloth cost at the same rate? (Change yards and meters to inches before making the statement.)

13. A and B engage in trade; A furnishes $400 capital, B, $600. They gain $150. What is the share of each in this gain?

SUGGESTION

whole capital : A's capital = whole gain : A's gain, etc.

14. The assessed valuation of a town is $3,500,000 and the tax to be raised is $4900. How much should a man pay of this tax who is assessed $7500?

15. A clock loses $\frac{1}{4}$ of a second every 5 minutes. How much will it lose in 12 hours?

16. How many pounds of coal can be bought for $3 when coal is $7.50 a ton?

17. How many rods in $\frac{27}{40}$ of a mile?

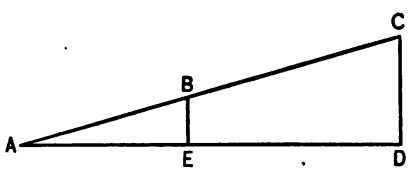
$$\frac{40}{40} : \frac{27}{40} = 320 \text{ rd.} : x \text{ rd.}$$

Suggestive Applications to Triangular Measurement

365. By geometry it has been proved that:

"The corresponding sides of similar triangles are proportional."

In the two similar triangles shown in the figure below, the side AE : side EB = side AD : side DC.



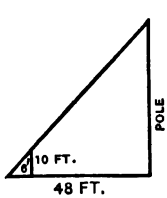
Upon this proportion, it is possible to base many practical measurements, and to calculate distances and heights not easily measured by the direct use of a pole or tape.

Suggestive Applications of Triangular Measurement, based upon the Proportion of the Sides of Similar Triangles

366. 1. Find the height of a pole which casts a shadow 48 ft. long at the time that a stick 10 ft. long casts a shadow 6 ft. long.

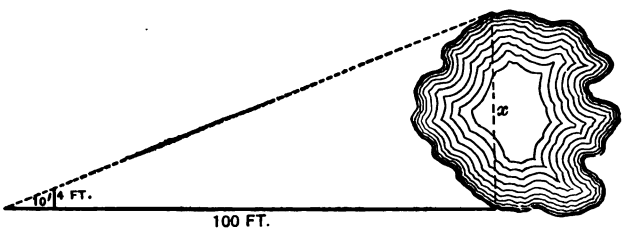
6 ft. : 10 ft. = 48 ft. : x ft.

x = height of pole.



2. Wishing to calculate the distance across a pond, I measured back from one of the points on the pond 100 ft. I then fixed a range from that

place to the other point, and measured across the two ranges 10 ft. from their place of meeting. I found it to be 4 ft. across from one range to the other at the point named. How far is it across the pond?



Let x = distance across the pond.
10 ft. : 4 ft. = 100 ft. : x ft.

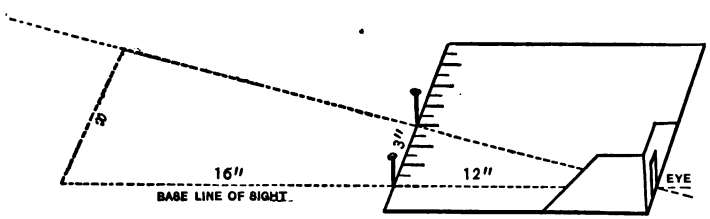
3. Had the distance across the ranges 10 ft. from their meeting point been 4 ft. 6 in., how far across would the pond have been?

A boy makes an instrument for calculating distances that he cannot easily measure in the direct way. He takes a large pasteboard box, and cuts away the sides, except at one corner. Through this corner he arranges a "peep hole" and in line with this hole, along the edge of the bottom, he marks a line 12 in. in length. Making a square corner at the farther end, he draws another line across the end of the bottom piece, and on this line he marks off a scale of inches and parts of inches.

He uses pins to fix his ranges on the line drawn

across the bottom piece, placing one pin at the corner, and the other where the range may be fixed as he looks at the object. In this manner, he can calculate the distance to an object when he knows its size, or the size of an object when he knows its distance from him. Study the description and the sketch, and see if you cannot make such a "measuring instrument."

The sketch shown below will show how the boy made his "measuring instrument." The materials are not important. Any one can make such an instrument from convenient materials, and test its accuracy by application. Many interesting and useful measurements can be made by such an instrument. It may be used to study the proportions of distant objects which one wishes to draw to scale, and can be used for either vertical or horizontal measurements. Care should be used in its use, that the line from pin to pin may be kept parallel to the distant line of measurement, or at right angles to the base line of sight.



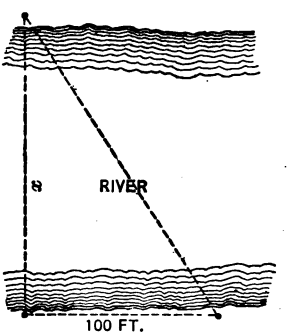
12 in. : 3 in. = 12 in. + 16 in. : x in.

Applications of the Use of "Measuring Instrument"

367. 1. Standing 50 feet from a barn, I measured it by use of the instrument shown in the sketch above. The range for its height separated the pins 6 inches, and the range for its width separated the pins $9\frac{3}{5}$ inches. Find the height and the width of the barn.

2. Wishing to know the width of a river, I measured a distance of 100 feet on its opposite bank, and set stakes. I then from the bank on the first side of the river made an observation, and found that the range separated the pins 8 inches. How wide was the river?

Let x = width of river.
Then $12 : 8 = x : 100$.

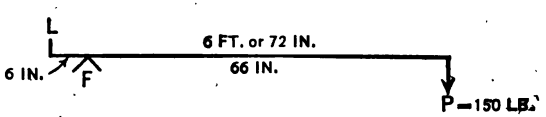


3. Wishing to know the height of a steeple, I took observations with the instrument, standing 120 feet from the base of the steeple. The range separated the pins 9 inches. How high was the steeple, allowance of 5 feet being made for the height of the "base line of sight"?

(By "distance," as used in the equation, is meant the distance from the fulcrum; "Power distance" meaning the distance of the power from the fulcrum.)

Suggestive Applications of the Use of the Lever

370. 1. A man bears down 150 lb. on the outer end of a bar of iron 6 ft. in length, which he has pushed under a heavy stone, and supported by a fulcrum 6 in. from the stone. How many pounds of load does he lift on the stone?



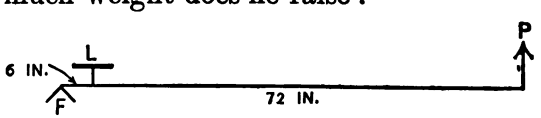
Statement. Let x = the load raised.

150 lb. $\times$ 66 = x times 6.

9900 lb. = 6 x.

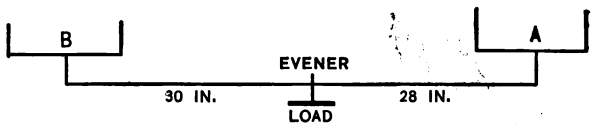
x = 1650 lb. *Ans.*

2. A man using a 6-ft. bar of iron for a lever, pushes its end 6 in. under a heavy weight and raises up on the outer end, using 150 lb. of power. How much weight does he raise?



3. As one of the horses of his team is much stronger than the other, a farmer adjusts the evener so that the stronger horse shall do

the harder work. It is adjusted as shown in the diagram.



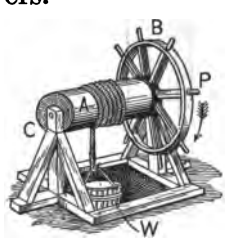
In a load of 2900 pounds how many pounds does each horse draw?

4. In a steelyard 24 inches long the distance from the weight hook to the fulcrum is 2 inches. How heavy a body can be weighed with a pound weight?

5. Two boys have a plank 18 ft. long, and wish to teeter, but one weighs 100 lb. and the other only 50 lb. Where will they fix the support that they may balance?

371. The wheel and axle and the windlass or capstan are but applications of levers.

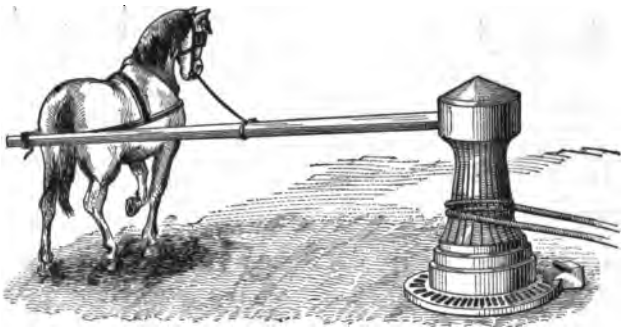
Power × length of crank = load × *radius* of the axle.



1. The diameter of the wheel, in a wheel and axle machine, is 8 feet, the diameter of the axle being 6 inches. What weight can be raised by a power of 100 pounds?

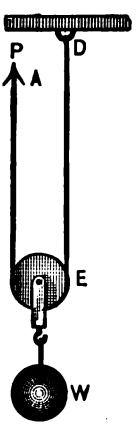
2. A horse pulls 100 pounds at the end of a lever 20 feet long, which turns an axle having a radius

of 6 inches. What load is drawn on the rope around the axle?

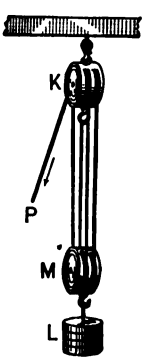


372. The pulley is used either to change the direction of the pull or to increase its force.

CHANGE OF DIRECTION
P:W = 1:1.



INCREASE OF FORCE
P:W = 1:2.



P:W = 1:4

373. To calculate the increase of force by the use of pulleys, we may use the equation:

Power × the number of separate parts of the rope that sustains the weight = the force or load.

1. A man wishing to raise a piano to the second-story window, adjusts the pulleys so that the weight is sustained by 4 parts of the rope. If the piano weighs 1000 pounds, how much power will it take to raise it when these pulleys are used?

374. The inclined plane is used to raise or lower heavy loads, by rolling them up or down an incline.

To calculate the gain by its use we may use the proportion:

Power : load = height raised : length of incline.

1. A man wishing to load a barrel of flour weighing 208 pounds (barrel included) into a wagon 3 ft. in height places a 12-ft. plank from the wagon to the ground and rolls the barrel up the incline. How many pounds of power does it take to move the barrel?

2. A man wishing to unload a barrel weighing 300 pounds from a wagon 3 ft. in height uses a 12-ft. plank as an incline, and also passes a rope around the barrel and fastens one end of the rope to the wagon, holding the other end. How much power does he use to hold the barrel as it rolls down the incline?

(The wagon holds $\frac{1}{2}$ of the load by this arrangement.)

Note. As the exercises in mechanical powers are only intended to be suggestive, further applications and the use of other powers is left to the teacher and pupils.

SUMMARY

Algebra deals with general applications, so that the solution of one problem forms a type for the solution of all similar problems. Some of its elementary principles may well be used, however, to aid in the solution of the problems which may also be solved by arithmetic.

In the solution of a problem by algebra, we first form a statement from the terms of the problem, and in the form of equations which show the relation of the given to the required terms. Then from these equations we find the value of the required terms.

Transposition is changing a quantity from one side of the equation to the other. This may be done by changing its sign as we pass the equation sign.

Equations having terms expressed in fractional form may be cleared of fractions by multiplying each term by the least common multiple of the denominators of the fractions.

When there are two or more unknown quantities in a problem, we must form from its conditions as

many independent equations as there are unknown terms used in the statements.

When there is more than one unknown quantity in the statements, we so combine the equations as to eliminate the unknown quantities by addition and subtraction.

Ratio is the relation of one quantity to another of the same kind. Ratio is expressed by the quotient of the first quantity divided by the second.

Ratio may be written by use of the colon, by use of the sign of division, or in fractional form.

Proportion is an equality of ratios. In a proportion the product of the extremes is equal to the product of the means.

The corresponding sides of similar triangles are proportional. By the use of this principle we may calculate heights or distances not easily measured in the ordinary way.

A simple measuring instrument may easily be made by which we may apply the principle of triangular measurement in finding the proportion of dimensions of distant objects, and in calculating heights and distances not easily measured in the direct manner.

The principles of leverage are of very common use. The law for calculating the effect of the use of a lever is:

Power × power distance = weight × weight distance.

The wheel and axle, the windlass and capstan, are applications of the lever. The law for calculating their effect is:

Power × length of crank = load × radius of axle.

The pulley may be used either to change the direction of the pull or to increase its force. The law governing its effect is:

Power × number of separate parts of the rope that sustains the weight = the weight.

The inclined plane is used to raise or lower heavy loads by passing them up or down an incline. The law for calculating the gain by its use is:

Power : load = height raised : length of incline.

CHAPTER XV

SQUARE ROOT

375. If a square field has a side of 4 units, its area is 16 square units. 16 is therefore called the *square* of 4, and 4 the *square root* of 16.

376. Find the squares of the following:

1. 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18.

2. .2, .03, .004, .0005, .0125, .025, .875, .75, .125.

3. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$.

4. 2345, 8076, 785, 5032, 45.25, 320, 2.003, 4.05, 99, 77.

377. From a study of the results in the last article write the square roots of the following:

1. 16. *Ans.* 4. Proof: $4 \times 4 = 16$. (Prove every answer.)

2. 64, 81, .25, .0049, .0016, 625, 8100.

3. $\frac{4}{9}$, $\frac{16}{25}$, $\frac{64}{81}$, $\frac{36}{49}$, $\frac{49}{64}$, $\frac{64}{100}$, $\frac{18}{144}$.

4. 121, 169, 196, 225, 289, 361, 256, 324, 625, 676.

TERMS USED IN SQUARES AND SQUARE ROOT

378. 1. The square of a quantity is indicated by writing a small figure 2 at the right-hand *upper* corner of the symbol for the quantity, thus 2^2, x^2, $\overline{.7854}^2$ etc. This figure, thus used to indicate the number of times the quantity is used as a factor, is called an *exponent.*

2. To indicate that the root of a quantity is to be taken, the *radical sign* is used as follows:

$$\sqrt{16},\ \sqrt{x},\ \sqrt{2500},\ \sqrt{16+9},\ \text{etc.}$$

To find the Square Root of Numbers, when it cannot be readily Seen

Note. Full explanation, step by step, of the "rule" for finding square root is often given. It does not seem necessary to do this, as the universal application of the plan is so easily tested, and every result so easily proved, that it seems wise to accept it *for the results* which it gives.

379. The following plan may be used to find the square root:

1. Point off the number into periods of two figures each, beginning at *units'* place.

2. Find the greatest root of the left-hand period and write it as the first figure of the answer.

3. Subtract the *square* of this first root figure from the left-hand period, and to the remainder annex the next *period.*

4. *Double* this root figure, and see how many

times the dividend, *exclusive* of its right-hand *figure*, will contain the result.

5. Write the number of times as the next answer figure, and *also* at the *right* of the trial divisor.

6. Proceed as in division and repeat the processes until the required root is found.

To *prove* the result, multiply the answer by itself, to see if it produces the original number.

Applications of Square Root

380. 1. Find the side of a square field which has an area of 2809 square feet.

CONVENIENT ARRANGEMENT OF WRITTEN WORK

$$\begin{array}{rl} 28'09(53 & 53 \text{ ft. } \textit{Ans.} \\ 25 & \\ \hline 103)3\,09 & \text{Proof: } 53 \times 53 = 2809. \\ 3\,09 & \\ \hline \end{array}$$

2. Find the side of a square field which has an area of 6889 square yards.

3. What is the distance around a square field which contains 6241 square yards?

4. A box which is square on the bottom and 5 feet in height contains 80 cubic feet. What is the length of the sides?

5. The area of a circle is equal to the square of its diameter multiplied by .7854. Find the diameter of a circle which has an area of 19.6350 square feet.

6. What will it cost to fence 10 acres in the form of a square, if the fence costs 25¢ a rod? (Change the 10 acres to square rods; then draw the figure and work the problem.)

381. Find the square root of the following and *prove* each answer:

1. 24336.	5. 30976.	9. 52.2729.
2. 190969.	6. 401956.	10. 872.6783.
3. 44521.	7. 93025.	11. 86.9432.
4. 743044.	8. 15625.	12. 60.3729.

13. 4.347225. 14. 43.267.

382. In finding the square root of fractions, we may

1. Extract the root of the numerator and the denominator *separately*, writing the results as the new numerator and the new denominator,

or

2. Change the fraction to decimal form and extract the root of the result.

For example:

$$\sqrt{\frac{4}{25}} = \frac{2}{5} \text{ or } .4, \text{ or } \frac{4}{25} = .16. \quad \sqrt{.16} = .4.$$

Caution. When the root does not come even, annex ciphers and carry out the answer to as many decimal places as may be desired. *Do not* use the remainder over the last partial divisor as if you were working in ordinary division.

383. Find the square roots of the following, carrying the roots to two decimal places when necessary:

1. $\frac{256}{289}$.	6. .7854.	11. $\frac{7}{8}$.	16. $\frac{3}{8}$.
2. 8.	7. 3.6.	12. .0081.	17. $\frac{9}{64}$.
3. $\frac{1}{5}$.	8. .9.	13. $6\frac{3}{4}$.	18. .400.
4. 6.	9. 4.25.	14. $8\frac{1}{2}$.	19. 81.04.
5. 2.5.	10. 7.	15. 12345.	20. 36.25.

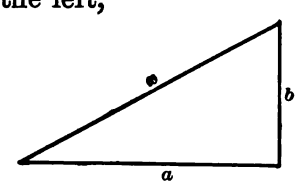
384. In a right-angle triangle the side opposite the right angle is called the *hypotenuse.*

385. The *square* of the hypotenuse of a right triangle is equal to the *sum* of the *squares* of the other *two sides.*

In the triangle shown, at the left,

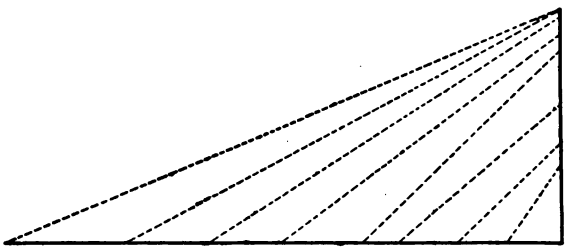
$$x^2 = a^2 + b^2.$$

Therefore, $x = \sqrt{a^2 + b^2}$.



This principle may be very easily tested as to its truth by measuring any convenient distance each way from the *square* corner of a piece of paper, board, top of desk, or any convenient object, and then squaring the numbers showing the distances which you measured, and extracting the square root of their sum. Test this result by comparison with the *measured* distance across the corner.

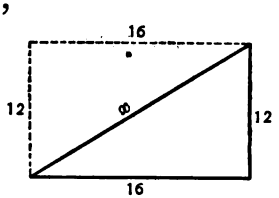
Measure sides and compute hypotenuse of several sets of triangles about a *square* corner, as suggested by the figure below. Compare the result which



you get by the computation with the actual distance across the corner as found by measurement. By this important principle, when we have given *any* two sides of a right triangle, we can compute the third side, for if

$$x^2 = a^2 + b^2,$$
$$x^2 - a^2 = b^2,$$
$$x^2 - b^2 = a^2.$$

In the problem, "Find the diagonal of a room 12 ft. by 16 ft.,"



$12^2 = 144.$ $16^2 = 256.$

$$\begin{array}{r} 144 \\ +256 \\ \hline 400 \end{array}$$

$\sqrt{400} = 20.$ 20 ft. *Ans.*

386. Problems involving the applications of squares to the sides of right triangles. (Make diagram for each problem.)

1. The diamond of a baseball field is 90 feet on a side, and has square corners. How far is it from home plate to second base?

2. The two sides of a right triangle are 57 inches and 76 inches. Find the hypotenuse.

3. A 40-ft. ladder leans against the side of a building, the base of the ladder standing 6 ft. from the base of the building. Find the height on the building at which the top of the ladder rests.

4. How long a rope will it take to reach from the top of a 60-ft. pole to a point 20 ft. from its base, if the pole stands erect on level ground?

5. A football field is 110 *yards* long and 160 *feet* wide. Find the length of its diagonal in feet.

6. The perpendicular of a right triangle is 72 feet and the hypotenuse 75 feet. Find the base.

7. Find the diagonal of a room 18 feet square.

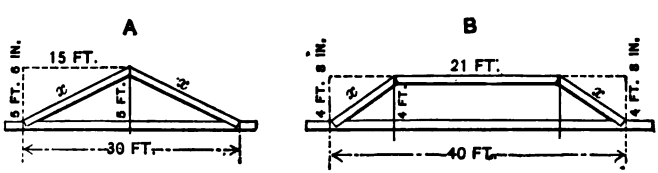
8. A diagonal path crosses near the square corner of a field. One end of the path is 60 ft. from the corner on one side, and the other end 80 ft. from the corner on the other side. How much farther is it around the corner than it is across it by the diagonal path?

9. How long a brace will it take to reach across a square corner from points 3 ft. and 4 ft. from the corner, measured on the sides which form the corner?

10. A man wishing to lay out a cellar has no convenient way of making the corners square. If he should measure 8 ft. on a line along one side, and 6 ft. on the line of the end, how far should he make it across from these points, that the corner may be square?

11. If the gable end of a barn, which is 30 feet wide, rises 10 feet, how long must the rafter be? (The rafter is the timber which supports the roof, reaching from the side and forming the slanting line of the gable.)

12. Two ships sail from the same point; one goes due north 128 miles and the other due east 72 miles. How far apart are the ships?



13. Country bridges are often built in the forms shown in the diagram. From the measurements given on the diagrams, find the length of the slanting timbers.

MEASUREMENTS

Note. For measurement of quadrilaterals, triangles, circles, etc., see pages 114 to 120.

Measurement of Solids

387. A solid is an object having length, breadth, and thickness.

1. The dimensions of a solid are given in linear units.

2. The surface of a solid is measured in square units.

3. The volume of a solid is measured in cubic units.

388. A prism is a solid having sides which are parallelograms and ends which are equal.

A prism having a small altitude in comparison to its other dimensions is often called a plinth.

A brick standing on end would be called, as to form, a prism.

A brick lying on its side would be called, as to form, either a prism or a plinth.

389. The altitude of a solid is the perpendicular distance between its bases.

390. A cylinder is a solid bounded by a uniformly curved lateral surface, and by parallel circles for its bases.

An unsharpened lead pencil when new, if it has circular ends, is in form a cylinder. If the sides of the pencil have been flattened, so that the ends

have straight lines in their perimeters, the pencil has the form of a prism.

391. A solid having a flat surface for its base, and sides which taper to a point opposite to its base, is called a *pyramid* if the base is bounded by straight lines, and a *cone* if the base is a circle.

The sides of a pyramid are formed by triangles. The altitude of these triangles is called the *slant height* of the pyramid..

392. A cube is a solid bounded by six equal squares.

393. A sphere is a solid bounded by a curved surface every point of which is equally distant from the center of the sphere.

1. A sphere is often called a ball.

2. A *great circle* of a sphere is a circle the plane of which passes through the center of the sphere.

Note. As type solids are common to every class room, and are so generally used in form study and drawing, it has not been considered necessary to introduce cuts of these solids in this consideration of their measurement.

Measurement of Surfaces of Solids

394. To find the *lateral* surface of a prism or a cylinder, multiply the perimeter of the base by the altitude.

395. To find the *entire* surface of a prism or a cylinder, find the sum of the lateral surface and the area of the bases.

Applications and Problems

396. 1. Find the lateral surface of a cylinder 5 feet in circumference and 4 feet in height.

2. Find the lateral surface of a cylinder 2 feet in diameter and 5 feet in altitude.

3. Find the entire surface of a square prism whose base is 4 by 4 inches and altitude $6\frac{1}{2}$ inches.

4. Find the entire surface of a cylinder 4 inches in diameter and $6\frac{1}{2}$ inches in length.

5. How many square inches of sheet iron will it take to make a length of stovepipe 6 inches in diameter and 30 inches long, if $\frac{3}{4}$ inch be allowed for the joint along the side of the pipe?

6. What will it cost to paint the side and bottom of a tank cylindrical in form, 10 ft. in diameter and 8 ft. in height, the work costing 12¢ a square foot?

397. The sides of a pyramid are formed by triangles, the sum of whose bases form the perimeter of the base of the pyramid, and whose altitudes are the slant height of the pyramid. The sides of a cone may be considered as formed by an infinite number of triangles.

The measurement of the lateral surface of the pyramid and the cone is, therefore, based upon the measurement of triangles.

398. To find the lateral or convex surface of a pyramid or a cone, multiply the perimeter of the base by one half the slant height.

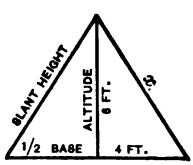
Applications and Problems

399. 1. Find the lateral surface of a rectangular pyramid whose base is 8 feet square and the slant height 6 feet.

2. Find the lateral surface of a cone whose base is 8 feet in diameter and whose slant height is 6 feet.

3. Find the *entire* surface of a pyramid whose base is 8 feet square and its slant height 6 feet.

4. Find the entire surface of a cone whose base is 8 feet in diameter and its slant height 6 feet.



5. Find the entire surface of a pyramid whose base is 8 feet square and its *altitude* 6 feet.

6. Find the entire surface of a cone whose base is 6 feet in diameter and its *altitude* 10 feet.

7. Find the convex surface of a cone whose base is 10 feet in diameter and its altitude 12 feet.

400. The surface of a sphere is four times the area of its great circle. To find the area of the surface of a sphere, *either* of the following methods may be used:

1. Multiply the area of its great circle by 4.

2. Square its diameter and multiply by 3.1416.

Applications and Problems

401. 1. Find the surface of a sphere 10 inches in diameter.

2. Find the area of the surface of a baseball 3 inches in diameter.

3. Find the *lateral* surface of a hemisphere resting on its flat surface, which is 6 inches in diameter.

4. Find the *entire* surface of a hemisphere which has a *radius* of 5 feet.

5. Find the cost of gilding a sphere 20 inches in diameter, at 15¢ a square foot.

Measurement of Volume of Solids

402. The volume of a prism or cylinder is equal to the area of the base multiplied by the altitude.

403. The volume of a pyramid or cone is equal to the area of the base multiplied by *one third* of the altitude.

404. The volume of a sphere is .5236 of the volume of a cube of like diameter.

Applications and Problems

405. 1. Find the volume of a prism whose base is 4 by 5 inches and its altitude 12 inches.

2. Find the volume of a cylinder whose base is 10 inches in diameter and its altitude 15 inches.

3. Find the volume of a pyramid whose base is 4 by 5 inches and its altitude 12 inches.

4. Find the volume of a cone whose base is 10 inches in diameter and its altitude 15 inches.

5. Find the volume of a 10-inch cube.

6. Find the volume of a 10-inch sphere.

7. A conical pile of coal has a circumference of 12.57 feet and an altitude of $3\frac{1}{2}$ feet. Allowing 35 cubic feet to the ton, find how many tons of coal there are in the pile.

8. How many gallons of water will a kitchen boiler hold which has the form of a cylinder and is 12 inches in diameter and 50 inches in height?

9. Find the weight of a sphere of cast iron 8 inches in diameter (cast iron weighs about 450 lb. to the cubic foot).

10. Allowing 490 lb. to the cubic foot, find the weight of a steel rod 3 in. in diameter and 10 ft. in length.

SUMMARY

Square Root

The square of a number is that number multiplied by itself.

The square root of a number is one of the two equal factors of the number.

The number of times a factor is to be repeated is often indicated by a small figure, called an expo-

nent, written at the upper right-hand corner of the factor.

The square root of a number is indicated either by the use of the radical sign or by the use of a fractional exponent $\frac{1}{2}$.

Right Triangles

In a right triangle the side opposite the right angle is called the hypotenuse.

The sides which form the right angle are often called the base and the perpendicular.

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solids

A solid is an object having length, breadth, and thickness.

A prism is a solid having sides which are parallelograms and bases which are equal.

The altitude of a solid is the perpendicular distance between the sides which are considered as its bases.

A cylinder is a solid bounded by a uniformly curved lateral surface, and having parallel circles for its bases.

A pyramid has a flat surface for one base, a point for the other, and triangular sides.

A cone has a circle for one base, a point for the

other, and sides bounded by a uniformly curved surface.

A sphere is a solid bounded by a curved surface, every point of which is equally distant from its center.

Surfaces of Solids

The lateral surface of a prism or cylinder is equal to the perimeter of the base multiplied by the altitude.

The entire surface of a prism or cylinder is equal to the lateral surface plus the area of the bases.

The lateral surface of a pyramid or cone is equal to the perimeter of the base multiplied by one half of the slant height.

The entire surface of a pyramid or cone is equal to the lateral surface plus the area of its base.

The surface of a sphere is equal to the square of its diameter multiplied by 3.1416.

Volumes of Solids

The volume of a prism or cylinder is equal to the area of the base multiplied by the altitude.

The volume of a pyramid or cone is equal to the area of the base multiplied by one third of the altitude.

The volume of a sphere is .5236 of the volume of a cube of like diameter.

GENERAL REVIEW EXERCISES AND PROBLEMS

406. In the following general review one exercise or problem or more is given for each essential principle of arithmetic.

1. Find the sum of the prime numbers between 1 and 50.

2. Find the difference between the sum of the even and the sum of the odd numbers from 25 to 75 inclusive.

3. Write the prime factors of 48.

4. Find the least common multiple of 15, 12, 24, 16, and 36.

5. Write by the Roman notation, 64, 375, 1910.

6. Divide 17 by .004.

7. Divide .003 by 14.

8. Divide $\frac{3}{8}$ by .05.

9. Divide .025 by $\frac{5}{8}$.

10. Add $16\frac{1}{4}$, .0125, $\frac{7}{8}$, 15.075, and $3\frac{1}{2}$.

11. From $\frac{3}{4}$ subtract .125.

12. From .875 subtract $\frac{7}{16}$.

13. Divide .875 by $\frac{7}{16}$.

14. From $145\frac{2}{3}$ subtract $117\frac{7}{9}$.

15. Multiply $1875\frac{2}{3}$ by 450.

16. Multiply $785\frac{5}{9}$ by $17\frac{5}{8}$.

17. If $\frac{3}{8}$ of a certain thing costs \$50, what will .25 of it cost?

18. If 875 pounds cost 30 ¢, what will $\frac{7}{16}$ pound cost?

19. If $\frac{5}{8}$ bushel cost 45 ¢, how many bushels can be bought for $36?

20. If 3.75 tons of coal cost $22.50, how many tons can be bought for $14?

21. A man owning $\frac{3}{8}$ of a factory sold .4 of his share for $6720. At the same price find the value of the part which he still owns.

22. A man owning $\frac{3}{8}$ of a factory sold 40 % of his share for $6720. At this price find the value of the whole factory.

23. Find the difference between .75 of $\frac{5}{8}$ and $\frac{4}{5}$ of .625.

24. After selling $\frac{1}{4}$ of his farm to one man and $\frac{2}{3}$ of the remainder to another, a farmer has 7.5 acres left. How many acres did he sell to the first man?

25. $\frac{1}{2}$ of a farm is worth $250 more than .375 of it. What is the value of the whole farm?

26. Find the sum of $678.45, $7869.50, $98762.30, $76289, $245, $76845.95, $67328.45, $786.75, $96875, $78645.25.

27. Divide $67845\frac{2}{3}$ by 6.

28. Find the cost of 760 pounds of coal at $6.50 a ton.

29. Sold three city lots for $1500, $2976, and $1895, respectively. What was the average price per lot?

30. Divide $248\frac{1}{3}$ by 48.

31. Divide 180 by $\frac{8}{15}$.

32. Divide $1\frac{14}{91}$ by $\frac{35}{52}$.

33. Find the value of $\frac{\frac{2}{3} \text{ of } \frac{11}{12}}{\frac{1}{18} \text{ of } 5\frac{1}{2}}$.

34. Find the value of $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{3}+\frac{3}{8}}$.

35. What part of $\frac{3}{4}$ is $\frac{1}{2}$?

36. If from $18\frac{3}{4}$ yards of cloth $2\frac{7}{8}$ yards are cut, what part of the whole is taken?

37. A can do a piece of work in 8 days and B can do it in 6 days. How long will it take them to do the work if they work together?

38. If $\frac{3}{4}$ of 9 bushels of oats cost \$13.50, what will $\frac{7}{8}$ bushel cost at the same rate?

39. Change $\frac{1}{32}$ to an equivalent decimal.

40. Change .9375 to an equivalent fraction.

41. Find the cost of 536700 bricks at \$12.50 per M.

42. Find the cost of 6785 pounds of coal at \$6.50 a ton.

43. At 75¢ a rod what will it cost to dig a trench $\frac{1}{4}$ mile long?

44. Reduce 54 yd. 2 ft. 10 in. to inches.

45. How much will 5 lb. 9 oz. 14 pwt. of gold dust bring at \$.75 a pwt.?

46. How many rods of fence will it take to inclose a field $\frac{3}{4}$ mile square?

47. Find the area in square yards, and the cost of fencing, at 75 ¢ a yard, a field which is 50 yards wide and 80 yards long.

48. Reduce 103720 liquid pints to higher denominations.

49. Reduce $\frac{1}{112}$ of a bushel to the fraction of a quart.

50. Reduce $\frac{2}{3}$ of a peck to the fraction of a bushel.

51. What decimal of a ton is 8 pounds?

52. What cost 2 bu. 3 pk. 6 qt. of peas at 30 ¢ a peck?

53. The war of the American Revolution began April 19, 1775, and the American Civil War began April 11, 1861. What length of time between these dates?

54. In 9 barrels of walnuts, each containing 2 bu. 3 pk. 6 qt., how many bushels?

55. Divide 202 yd. 1 ft. 6.75 in. by $\frac{3}{5}$.

56. Divide 311 gal. 1 qt. 1 pt. by 53.

57. When it is 3 o'clock at Washington, what is the time in Chicago? In San Francisco? In Boston?

58. When it is noon in London, what is the time in New York?

59. A ship's chronometer, keeping London time, shows 1:35 P.M. when it is noon by the sun. What is the longitude of the ship?

60. The longitude of Chicago is 87° 37′ 45″ W. and the longitude of Paris is 2° 20′ E. What is the difference in sun time in the two places?

61. What is the length of a lawn that contains 305 sq. yd. and is 45 ft. wide?

62. How many square yards in the side walls and ceiling of a room that is 15 ft. wide, 20 ft. long, and 9 ft. high?

63. A rectangular farm is 189.5 rd. long and 150 rd. wide. What is its value at $75 an acre?

64. A rectangular farm contains 435 acres, 96 square rods, and is 264 rods on one side. Find its dimensions and the cost of inclosing it by a fence that costs 25¢ a rod.

65. What is the capacity of a bin that is 8 ft. 6 in. by 6 ft. by 4 ft. 4 in.?

66. A vat having a square bottom 12 ft. on a side contains 1224 cu. ft. What is its height?

67. What will it cost to dig a cellar 45 ft. long, 28 ft. wide, and 6 ft. deep at 40¢ a cubic yard?

68. How many board feet in a board 1 in. thick, 14 ft. long, and 8 in. wide?

69. Find the cost of 10 planks, each 16 ft. long, 15 in. wide, and 3 in. thick, at $ 25 per M.

70. What will the lumber for a floor 12 ft. by 20 ft. cost if the flooring is 1 in. thick, and $\frac{1}{8}$ is allowed for matching, the cost of the lumber being $30 per M?

71. How many bushels will a bin hold whose inside dimensions are 6 ft., 5 ft., and 4 ft.?

72. How many tons of ice can be packed in a space 40 ft. by 30 ft. by 20 ft., a cubic foot of ice weighing $58\frac{1}{8}$ lb.?

73. How many gallons of water will a cistern hold which is 4 ft. by 5 ft. by 6 ft.?

74. How many gallons of water will a cistern in the form of a cylinder hold, if its dimensions are: diameter 5 ft. and depth 6 ft.?

75. How many ounces of gold are equal in weight to 10 lb. of lead?

76. A farmer sold 3150 bushels of grain and had 30 % of his crop left. What was the entire crop?

77. A merchant sold a suit of clothes for $18, which was $12\frac{1}{2}$ % more than it cost him. What did the suit cost the merchant?

78. A man sold two houses at $2500 each. On one he gained 20 % and on the other he lost 20 %. Find his total gain or loss.

79. Bought paper at $3 a ream and sold it at 25¢ a quire. What was the gain per cent?

80. A's money is 20% more than B's. What per cent is B's less than A's?

81. An architect charged $\frac{1}{4}\%$ of the cost of a building for plans and specifications, and $1\frac{1}{2}\%$ for superintending the construction of the building. What was his total commission, if the building cost $6000?

82. Find the interest at 6% on $575 for 6 mo. 18 da.

83. Find the interest at 6% on $500 from April 10, 1908 to March 5, 1909.

84. Find the amount due on a note at its maturity which was given March 3, 1910, for 4 months, for $125 at 6%.

85. What is the present worth and the true discount at 6% of a note without interest which is due in 90 days, and whose face is $150?

86. What is the bank discount and proceeds of a note without interest due in 90 days and written for $150?

87. Find the cost of 200 shares of railroad stock the market value of which is $124\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

88. Find the cost of insuring a house for $5000 at $\frac{3}{4}\%$.

89. Find the tax on property assessed for $4500, the rate being 1.55408 per centum.

90. Find the net cost of goods listed for $450, discounts of 25%, 20%, and 10% being allowed.

91. What single discount is equivalent to discounts of 20%, 10%, and 5%?

92. What is the duty on $100 worth of musical instruments, the rate being 45%?

93. What is the duty on 25 gross of lead pencils listed at $3 per gross, the rate being 45 cents a gross specific, and 25% ad valorem?

94. What is the cost of a sight draft on New York for $500, the rate of exchange being $\frac{1}{2}$% premium?

95. Bonds selling at $89\frac{7}{8}$ pay 4% dividends. What per cent on his investment will a man make who buys these bonds, paying $\frac{1}{8}$% brokerage?

96. Find the cost of a bank draft for $500 when the rate of exchange is $\frac{1}{2}$% premium and the bank charges $\frac{1}{10}$% for its labor, etc.

97. Find the cost of a demand draft on London, exchange being 4.8525, the face of the draft being £100.

98. Find the cost of a sight draft on Paris for 500 francs, exchange being 5.18.

99. Change the following to longitude measure: 3 hr. 12 min. 30 sec.

100. Change the following to time measure: 12° 15′ 45″.

101. Reduce 75 m. 24 cm. to centimeters.

102. Change 750 km. to miles.

103. Express in metric terms: 5 ft.; 2 mi.; 4 in.; 20 yd.

104. Express in our measure: 5 km.; 30 m.; 24 cm.

105. Find the cost of 40 acres of land at $250 a hektare.

106. How many acres in a field that contains 50 hektares?

107. How many cubic meters of water will a tank hold that has inside dimensions as follows: length 2 m., width 1 m. 40 cm., depth 1.75 m.?

108. How many cubic yards of earth will have to be removed to dig a cellar 8 m. long, 4 m. wide, and 1.75 m. deep?

109. How many hektoliters of grain will a bin hold whose inside dimensions are 2.50 m. long, 2 m. wide, and 2.40 m. deep?

110. How many bushels of grain in 50 hektoliters?

111. How many liters in 300 quarts of water?

112. How many liters in 300 quarts of grain?

113. Find the cost of 500 pounds at $1 a kilogram.

114. Express 50 kilograms as U. S. pounds.

115. Find the weight of water in a water tank in the form of a cylinder whose diameter is 1.50 meters and length 3 meters.

(1 cu. dm. = 1 kilogram = 2 $\frac{1}{5}$ lb. av.)

116. Find the area of a circle whose diameter is 2.50 meters.

117. How much French money should I receive in exchange for $25 U. S. gold?

118. How much English money should I receive in exchange for $25 U. S. gold?

119. How much German money should I receive in exchange for $25 U. S. gold?

120. How much Holland money should I receive in exchange for $25 U. S. gold?

121. What centigrade reading is equivalent to 70 deg. Fahrenheit?

122. What Fahrenheit reading is equivalent to 30 deg. centigrade?

123. What Fahrenheit reading is equivalent to 10 deg. below zero centigrade?

124. Bought silk in Paris at 10 francs a meter, paid duty of 50 %, and sold it in this country at $3.85 per yard. Find the sum gained or lost per yard.

125. Find the value of x in the equation:

$$\frac{x}{2} - 3 + \frac{x}{3} = 2.$$

126. A rectangular field has a length which is 4 times its width. Its perimeter is 300 rods. Find its area.

127. A man paid $500 for two horses. One cost $\frac{2}{3}$ as much as the other. Find the cost of each.

128. Divide 30¢ between two persons so that one shall have 50 % more than the other.

129. Divide 45¢ between two persons so that $\frac{1}{3}$ of the greater amount shall equal $\frac{1}{2}$ of the less.

130. Find the values of x and y in:

$$10x - 4y = 16.$$
$$7x - y = 13.$$

131. Find the values of x and y in:

$$\frac{x}{8} + \frac{y}{15} = 13.$$

$$\frac{\frac{x}{2} + 8x + 2y}{10} = 4.27.$$

132. Divide 25 into 2 such parts that 3 times the less shall equal 2 times the greater.

133. What is the ratio of $\frac{1}{2}$ to $\frac{1}{4}$?

134. What ratio has 1 lb. Troy to 1 oz.?

135. Find the value of x in the proportion:

$$\tfrac{5}{8} : 625 = 45 : x.$$

136. Find the value of x in the proportion:

$$x : 9 = 8 : 6.$$

137. If $\frac{2}{3}$ of a farm cost \$750, what will $\frac{3}{8}$ of the farm cost? (Solve by proportion.)

138. A vertical pole 8 ft. in height casts a shadow 24 ft. long at the same time a church steeple casts

a shadow 345 ft. long. Find the height of the steeple.

139. A man places an iron bar which is 6 feet long so that the end is under the edge of a block of marble, and he then places a stone 6 inches from the end of the bar, and bears down on the outer end 150 pounds. How much weight does he lift at the other end of the bar?

140. In moving a building a man arranges a capstan so that a horse draws at the end of a lever 20 ft. long, and the rope winds around the axle, which is 5 in. in radius. How many times does he increase the power of the horse by this arrangement?

141. A man uses a plank 12 ft. long as an inclined plane that he may roll a barrel up into a wagon that stands 4 ft. in height. How many times does he increase his power by this arrangement?

142. Find the square root of 234256.

143. Find the square root of $6\frac{1}{9}\frac{}{3}$.

144. Find the square root of .007921.

145. Find the hypotenuse of a right triangle whose base is 6 inches and its altitude 10 inches.

146. Find the altitude of a right triangle whose base is 12 inches and its hypotenuse 15 inches.

MISCELLANEOUS PROBLEMS FOR ADVANCED CLASSES

407. 1. One half the sum of two numbers is 800, and one half the difference of the same numbers is 200. What are the numbers?

2. What time is it when the time past noon is $\frac{3}{4}$ of the time to midnight?

3. 20% of $\frac{3}{8}$ of a number is what per cent of $\frac{3}{5}$ of it?

4. James is 51 miles in advance of Frank. James travels 16 miles per hour and Frank travels 19 miles per hour. In how many hours will they be together if they travel in the same direction?

5. What number is that which, increased by its half, its third, and 18 more, will be doubled?

6. How many cubic yards of earth will be required to raise 9 in. the surface of a lawn which is 40 ft. long and 27 ft. wide?

7. How much more water can be discharged by a pipe which is 2 inches in diameter than by one which is $\frac{1}{2}$ inch in diameter?

8. How much money must a man send his agent that he may buy 300 barrels of flour at $5.75 a barrel, after deducting his commission of .25 %?

9. A man sent his agent $2500 to use in purchase of grain, after deducting his commission of $\frac{1}{2}$ %. How much money does the agent use in the purchase of the grain?

10. What is the rate per cent of profit on apples bought at 3 for 5¢ and sold at 2 for 5¢?

11. How far is it from the lower corner of a room to the opposite upper corner if the room is 12 by 14 ft. and 9 ft. in height?

12. The inside diameter of a ring is 2 feet, the outside diameter being 1 inch greater. The ring is $\frac{1}{4}$ inch thick. Find the number of cubic inches in the ring.

13. A man discounted a note for 90 days at 6% and received as proceeds $350. Find the face of the note.

14. Two successive discounts of 20% and 15% reduce a bill $306. How much was the original bill?

15. What decimal bears the same ratio to .05 that $\frac{3}{8}$ does to $1\frac{1}{2}$?

16. How long must $120 be at interest at 6% to earn $34.64?

17. The United States government pays exact interest at 5% from April 1 to Oct. 10 on a claim of $6000. Find the interest which the government pays.

18. How many cubic centimeters of lead are there in a piece of lead pipe 1 m. long, the outer diameter being 5 cm. and the thickness of the pipe 5 m.?

19. How many feet per second are equal to a speed of 40 miles per hour?

20. What is the difference in square rods between $\frac{3}{4}$ of an acre and $\frac{3}{4}$ % of an acre?

21. If a man can row $6\frac{1}{2}$ miles an hour downstream and $4\frac{1}{2}$ miles up the same stream, how far could he row in an hour in still water?

22. Find the length of the minute hand of a clock whose point moves 5 inches in 15 minutes.

23. When it is noon Jan. 2, 1910, in New York, what is the time and date at Luzon, longitude 120° East? (Use Standard Time.)

24. If it costs \$5 to travel 384 kilometers by rail, what is the rate of fare in cents per mile?

25. What is the rate of gain when 25 % of the selling price equals the gain?

26. If $\frac{1}{3}$ of an article is sold for $\frac{1}{3}$ of its cost, what is the rate per cent of loss?

27. For how much must I write a note that it may be discounted at bank at 6 % for 90 days and yield me \$500 as proceeds?

28. A board 1 inch thick is 14 inches wide at one end and 10 inches wide at the other, and its length is 16 feet. How many board feet of lumber does it contain?

29. A has \$60, B has \$40. What per cent of money has A more than B? What per cent has B less than A?

30. What is the difference between the true and the bank discount on \$500 for 6 mo. at 6%?

31. A bin will hold 50 bushels of oats. How many bushels of apples will it hold?

32. What is the smallest diameter of a log that will square 8 inches?

33. A man deposits \$250 in a savings bank Dec. 31, and leaves it for two years on interest at 4%. What sum will he then have to his credit in the bank, interest paid semiannually?

34. A man sold a horse for \$250, but was unable to collect 10% of the debt. Although he was unable to collect the full amount, he still made 5% profit. What was the value of the horse to the man who sold it?

35. How much longer wire will it take to wind 5 times around a stovepipe 5 inches in diameter than it will to wind 3 times around a pipe 6 inches in diameter?

36. What is the per cent of increase when a farmer raises 44 bushels of oats from 4 bushels of seed?

PRESTON-STEVENS STANDARD ARITHMETIC

ANSWERS

Art. 38

1. 1076197.
2. $1922.75.
3. 4137.42425.
4. 877,664,999.
5. $1885.23.
6. 2635.679.
7. $1292.22.

Art. 40

1. 95.
2. $399.50.
3. 18.55.
4. $7435.
5. 3,756,330.
6. 3269.
7. 95.5.
8. $9.
9. 1664.
10. $409.75.
11. 1194.
12. 35136.
13. 69585.
14. 6894.
15. 66465.

Art. 45

1. $75.
2. $25.
3. $750.
4. $4.
5. $675.
6. 37.5.
7. 40.
8. $2.50.
9. $150, $250.
10. $1175.
11. $75.
12. 2580.
13. 4220.
14. 825, 550.
15. 22.50.
16. 4.75.
17. 130.75.

Art. 50

1. $31.50.
2. $120.
3. 32240.
5. $28875.
6. $27.
10. $256.
11. $48.
12. $550.
13. $2240.

Art. 53

1. 2891064.
2. 6571362.
3. 3911434.
4. 27166272.
5. $31942.75.
6. $86088.
7. 22940.25.
8. 4262.82.
9. 74.35263.
10. 2.45455.
11. $1108.525.
12. $13113.1875.
13. 60 bu.
14. 37.5 A.
15. $250.
16. $47.05.

Art. 54

1. $2.82.
2. $97.50.
3. $767.50.
4. $14.25.
5. 8219.45.
6. 15.1420.

Art. 59

1. 1508, 20153, 2467, 25484, 292107, 201081⅔.
2. 16956, 11321½, 10802½, 10599, 114465¾, 114195, 750½.
3. 1346, 11573, 8643, 13578, 9152⅔, 86421⅕, 179208⅘.
4. 1130, 1310, 75536⅓, 70594⅔, 762720⅓, 7315937⅓.
5. 963⅓, 6474⅔, 6179⅔, 654133⅓, 62325, 1252074⅔.
6. 84755¼, 5820, 5665⅘, 59869¼, 572329, 5721116⅘.
7. 75357, 50363, 4865⅘, 50255⅔, 47335, 853691⅓.
8. $323\frac{1}{21}$, $2158\frac{2}{21}$, $2060\frac{8}{21}$, $2158\frac{8}{21}$, $2085\frac{4}{21}$, $2060\frac{8}{21}$.

9. $132\frac{4}{11}$, $15067\frac{1}{17}$, $8897\frac{1}{17}$, $847\frac{12}{51}$, $8305\frac{4}{17}$, $9339\frac{8}{51}$.
10. $942\frac{3}{8}$, $6009\frac{5}{12}$, $6359\frac{7}{24}$, $600\frac{3}{8}$, $5907\frac{1}{8}$, $5877\frac{7}{24}$.
11. $5327^{-}+154$ rem., $5328^{-}+414$ rem., $562681^{-}+352$ rem., $562^{-}+438$ rem., $4929^{-}+90$ rem.
12. $1267\frac{5}{29}$, $1562\frac{24}{29}$, $1492\frac{18}{29}$, $1562\frac{22}{29}$, $1512\frac{24}{29}$, $146055\frac{6}{29}$.
13. $476+621$ rem., $620+387$ rem., $60+136$ rem., $668+660$ rem.
14. $686+932$ rem., $5391+147$ rem., $4383+764$ rem., $43829+935$ rem.

Art. 63

1. .0005.
2. 5000.
3. .05.
4. .5.
5. 5000.
6. 5.
7. 50.
8. 500.
9. .00005.
10. .21.
11. .15.
12. 16.895+.
13. 6000.
14. 30.2.

Art. 66

1. 9125.
2. $98.
3. 137.
4. $602.
5. $704.25.
6. 18 min.
7. 72.
8. 26 yr.
9. 812.7912.
10. 30 hr.
11. $125.
12. 16.
13. $9000.
14. $12.50.
15. 91.87+.
16. 10.
17. 12.5 tons.
18. $231.25.
19. $3.35 or 5.317+ ct. per gal.
20. $38.69.
21. 19493.
22. $147.75.
23. 555.
24. .01527.

Art. 80

4. $2\frac{1}{7}$. 6. $6\frac{17}{18}$. 8. $5\frac{7}{13}$. 10. $27\frac{7}{9}$.
5. $1\frac{13}{14}$. 7. $6\frac{23}{24}$. 9. $8\frac{1}{17}$. 10. $27\frac{7}{9}$.

Art. 81

3. $\frac{25}{2}$. 5. $\frac{112}{3}$. 7. $\frac{124}{5}$.
4. $\frac{32}{3}$. 6. $\frac{44}{3}$. 8. $\frac{121}{3}$.

Art. 84

5. $\frac{5}{3}$. 9. $\frac{5}{12}$. 15. $\frac{1}{4}$. 19. $2\frac{7}{9}$.
6. $\frac{8}{3}$. 12. $1\frac{5}{7}$. 17. $2\frac{3}{4}$. 20. $3\frac{1}{3}$.
7. $\frac{7}{12}$. 13. $\frac{1}{4}$. 18. 2.
8. $\frac{1}{12}$. 14. $1\frac{3}{5}$.

Art. 85

3. $172\frac{3}{4}$. 4. $286\frac{3}{4}$. 5. $122\frac{1}{24}$. 6. $44\frac{7}{18}$.

Art. 86

4. $92\frac{5}{8}$. 7. $92\frac{1}{24}$. 9. $69\frac{40}{63}$.
5. $73\frac{5}{36}$. 8. $95\frac{21}{80}$. 10. $39\frac{187}{210}$.
6. $105\frac{1}{48}$.

Art. 87

2. $6\frac{1}{3}$. 10. $12\frac{79}{90}$. 15. $13\frac{3}{4}$.
3. $13\frac{1}{3}$. 11. $184\frac{3}{7}$. 16. $19\frac{3}{8}$.
6. $11\frac{11}{12}$. 12. $2\frac{13}{24}$. 17. $26\frac{1}{2}$.
7. $117\frac{7}{8}$. 13. $67\frac{5}{8}$. 18. $2\frac{3}{4}$.
8. $63\frac{29}{30}$. 14. $33\frac{7}{8}$. 19. $11\frac{1}{4}$.
9. $5\frac{11}{24}$.

Art. 89

5. $54\frac{1}{3}$. 6. $\frac{32}{45}$. 8. $\frac{3}{10}$. 9. $\frac{2}{5}$. 10. $\frac{4}{7}$.

Art. 92

1. $1\frac{1}{3}$.
2. $\frac{9}{14}$.
3. $\frac{25}{36}$.
4. 96.
5. $2\frac{23}{56}$.
6. $\frac{42}{64}$.
7. 3.

Art. 93

3. $\frac{4}{3}$.
4. 4.
5. $3\frac{3}{5}$.
6. $\frac{1}{3}$.
7. $10\frac{3}{5}$.
8. $2\frac{1}{7}$.

20. 10.12.
21. 4.6666+.
22. .83333+.
23. .8888+.
24. .73333+.
25. .77127+.
26. .4.
27. .85714+.

l+.

Art. 104

6. 11.221+.
7. .744+.
8. .992+.
9. 11.862+.
10. .888+.

Art. 107

9. $\frac{7}{24}$.
10. 24.
11. 14 d.
12. $47\frac{1}{3}$.
13. $\frac{1}{4}$.
14. $600.
16.66+. 15. $3\frac{3}{4}$.

s $\frac{1}{24}$ greater.

31. 102.
32. $625.
33. .0064.
34. $41\frac{3}{13}$.
35. $200.
36. 6.
37. .09375.
38. $1.33+.
39. $5000.
40. $4.50.
41. $\frac{3}{7}$.
42. $\frac{13}{24}$.
43. $2.34.

8. $65\frac{5}{8}$.
9. $11\frac{1}{16}$.
10. $4\frac{1}{40}$.
13. $\frac{4}{25}$.
14. $\frac{13}{16}$.
15. $4\frac{1}{3}$.
18. $5_{\frac{1}{125}}$.
19. $6_{\frac{1}{400}}$.
20. $15_{\frac{3}{1000}}$.
22. $50\frac{1}{4}$ m.
23. $\frac{16}{495}$.
24. $56.
25. $\frac{3}{4}$.
26. $3\frac{1}{13}$ da.
27. $\frac{5}{13}$ is $\frac{1}{140}$ gr.
28. $41\frac{7}{9}$.
29. 13.55.
30. $\frac{2}{9}$.
44. $1208.48.

Art. 103

1. .625.
2. .875.
3. .3125.
4. .4375.
5. .9375.
6. .5333+.
7. .41666+.
8. .078125.
9. .69444+.
10. .36.

45. $1\frac{17}{30}$ T. 3233$\frac{1}{3}$ lb.
46. $\frac{1}{8}$.
47. 660.
48. 666$\frac{2}{3}$ yd.
49. 54 gal.
50. $12800.
51. $4.62.
52. $990.
53. 18 yr., 12 yr.
54. 20 da.
55. $2500.
56. $2\frac{2}{5}\frac{2}{9}$ da.
57. $15.
58. $23.70.
59. $\frac{5}{24}$.
60. 18.
61. $2880.
62. $\frac{1}{3}$.
63. 18 yd.
64. $4.80.
65. $\frac{6}{11}$.

Art. 113

2. 218 qt.
3. $1.50.
4. 34.
5. 31.
6. 133.
9. 5 pk. 1 qt. 1 pt.
10. 122 pt.
11. 63 pt.
12. 925 qt.
13. 15 gal. 3 qt. 1 pt.
14. 191.
17. 13 gal. 1qt. 1 pt.
18. 3 bu. 2 pk. 71 pt.
20. 39 gal.
21. 152 bu. 1 pk. 6 qt.
23. 5 gal. 2 qt. 1.8 pt.
24. 2 qt. 1$\frac{1}{8}$ pt.
25. 2 pk. 7.75 qt.
28. 1 gal. 1$\frac{2}{4}\frac{3}{5}$ pt.
29. 12$\frac{17}{22}$.

Art. 120

1. $48.60.
2. 1599.
3. 49.984375.
5. 5 yr. 9 mo. 12 da.
6. 26 yr. 1 mo. 4 da.
7. 14° 15′.
8. $\frac{1}{4}$.
9. 40.99+.
10. 1806 lb. 15 oz.
11. $1.92.
12. 83 lb. 10 oz. 3 pwt. 3 gr.
13. 1 mi. 1660 yd. 9 in.
14. 25 da.
15. $1.80.
16. 34.
17. $800.
18. $81.
19. $.43.
20. $1.20.
21. 12 hr.
22. $.02875.
23. 123.2.
24. 15.
25. 375.
26. 21¢.
27. $25.50.
28. $3.70.
29. 333$\frac{1}{3}$.
30. 13 yr. 6 mo. 29 da.
31. .25.
32. $18.20.
33. $3.25.
34. 546.8.
35. 1610.
36. $40 gain.
37. $.75 gain.
38. 73 ft. 4 in.
39. $\frac{1}{3}$.
40. Aug. 12, 1911.
41. .3125.
42. $\frac{7}{8}$ bu.
43. 1056.
44. 18 T.
45. 11$\frac{2}{3}\frac{1}{3}$ yd.
46. 64.
47. 100.
48. $15.
49. 5° 15′ more.
50. 1 mi. 47 rd. 4 yd.

Art. 134

3. 360 sq. ft.
4. 440 sq. ft.
5. 576 sq. ft.

Art. 149

6. 4805 sq. rd.
7. 5 sq. yd. 8 sq. ft. 8 sq. in.
8. $\frac{1}{2}$.
9. $\frac{3}{8}$.
10. 200 rd. 15 A.
11. 156 ft.
12. $14850.
13. 370.
14. 15.708 ft.
15. $3187.50.
16. 444$\frac{4}{9}$ sq. yd., $140.
17. 40 rd.
18. 40 yd.
20. $10.
21. $80.
19. 217 sq. ft.
22. 18 rd. 2.025 A.
23. 43.9824 ft., 154 sq. ft. nearly.
24. 45.
25. Nearly 8 in.
26. 16 sq. ft., 3.4336 sq. ft.
27. 312 sq. yd.
28. $31.20.
29. $79.20.
30. 220.
31. 54.
32. 18.8496 ft.
33. 15.708 ft.
34. 168.4 times.
35. 12$\frac{5}{6}$.
36. 7500.

37. 2401 nearly. 38. $11.39.
39. Same.
40. 160 yd. and 200 yd.
41. 801. 46. 330.
42. 11. 47. $1388\frac{8}{9}$.
43. 4. 48. .21875.
44. 720, 1440. 49. 159+ ft.
45. 78.54 sq. ft.

Art. 151

3. 160. 5. $32.40. 7. $46.67.
4. 192. 6. 216.

Art. 152

2. $411\frac{3}{7}$. 4. $151\frac{1}{2}$. 6. 3172.5.
3. $171\frac{7}{8}$. 5. 280. 7. 325.

Art. 158

4. 60. 10. $45\frac{9}{35}$.
5. 86.7+. 11. 134.65.
6. 69+. 12. 11.75 in.
7. 1 cu. yd. 13. $27\frac{7}{8}$.
8. 720. 14. 2112.
9. 157+. 15. $6\frac{1}{4}$ ft.
16. Nearly 19 bbl.
17. 470+ gal. 22. 14.54 cu. yd.
18. 29.376 gal. 23. 2700 lb.
19. 1175. 24. 64+ lb.
20. 4712+. 25. 80 lb.
21. 16.7+.

Art. 165

1. 12. 10. $600. 18. $99.
2. $7.50. 11. $162. 19. $11.70.
3. 18 bu. 12. 300 bu. 20. $27.
4. 6. 13. $720. 21. $1.40.
5. 200. 14. $72. 22. $125.
6. 5250. 15. $7.20. 23. 60.
7. 80. 16. $3. 24. 32.
8. 120 A. 17. $.90. 25. 67.5.
9. 73.5. 26. $506.25.
27. 480. 32. $1\frac{1}{2}$ hr.
28. 57.5. 33. $360.
29. 97+ lb. 34. 540.
30. $56.25. 35. $50.
31. 432.

Art. 166

3. $1250. 19. 7500.
4. $6000. 20. 375.
5. $320. 21. $1200.
6. $4000. 22. $12000.
7. 2500. 23. $7500
8. $5450. 24. 20¢.
12. 950. 25. $5000.
13. 400. 26. $750.
14. 1440 lb. 27. $2850.
15. 62.5 lb. 28. $288000.
16. 490 lb. 29. 2450.
17. 200000. 30. $130.
18. 1250.

Art. 167

1. $\frac{1}{5}$, 20%. 9. 24%.
2. $\frac{1}{10}$, 5%. 10. $16\frac{2}{3}$%.
3. 25%. 12. 20%.
4. 31.25%. 13. 40%, 36%.
5. 10%. 14. 60%.
6. 10%. 16. $12\frac{1}{2}$%.
7. 60%. 17. 60%.
8. 13.89%, 720%. 18. $42\frac{85}{100}$%.
19. $82\frac{89}{100}$% nearly.
20. 65%.
21. $66\frac{2}{3}$%, 80%, $62\frac{1}{2}$%, 150%, 200%.
22. $12\frac{1}{2}$%. 23. $33\frac{1}{3}$%, 25%.
24. 20%, 40%.

Art. 168

1. 7.4+. 5. 320.
2. $4000. 6. $360.
3. 4800. 7. $6\frac{2}{3}$.
4. 12. 8. 32.

9. 594.
10. 224.
11. $66\frac{2}{3}$.
12. 80.
13. $27\frac{3}{11}$.
14. 1000.
15. $\$555\frac{5}{9}$.
16. 700.
17. 40.
18. 45.
19. 42+.
20. $4\frac{65}{100}$.
21. $78\frac{54}{100}$.
22. 31.8+.
23. 125.

Art. 175

1. $18.75.
2. $25.
3. $337.50.
4. $27.
5. $55.
6. $132.15.
7. $12187.50.
8. $23.75.
9. Com. $281.25, pro. $10968.75.
10. $10.
11. $5.63.
12. $2\frac{1}{2}\%$.
13. Com. $136.20, net profit $223.80.
14. $3.
15. $27540.
16. $4363.75.
18. $995.02+.

Art. 179

2. $3.80.
3. $648.
4. $26.60.
5. $513.
6. $5.
8. $83\frac{1}{3}\%$.
9. $161.50.
10. $102.60.
11. Single discount $1.73.
12. $15, 25%.

Art. 188

1. $99.65.
2. $83.92.
3. $174.08.
4. $6.25.
5. $5\frac{1}{4}$ mills.
6. $31.50.
7. .0054945.
8. $5.40.
9. $112.50.
10. .00174842.
11. $16.50.
12. $19.
13. $6150.
14. $25.
15. 1.1922.
16. $1.11.
17. $22458.

Art. 198

1. $198.17.
2. $301.55.
3. $260.55.
4. $5.
5. $8.
6. $7.50.
7. $12.
8. $60.
9. $22.
10. 222 francs.
11. $40.
12. $33.78.
13. $9.65.
14. $12.17.
15. $7.01.
16. $5.50.
17. $10.
18. $37.50.
19. $15.
20. 30¢.
21. 5¢.
22. $33.

Art. 205

2. $40.
3. $5.
4. $3.50.
5. $120.
6. $36.25.
7. $28.75.
8. $27.30.
9. $90.10.
10. $9.59.
11. $1.44.
12. $12.
13. $11.84.
14. $12.33.
15. $82.19.

Art. 207

1. $11.11.
2. $38.68.
3. $75.75.
4. $22.56.
5. $26.42.
6. $11.90.
7. $56.25.
8. $110.
9. $54.31.
10. $41.07.
11. $122.50.
12. $145.06.
13. $29.60.
14. $79.20.
15. $13.80.
16. $25.82.

Art. 209

2. $49.
3. $51.67.
4. $38.50.
5. $78.75.
6. $64.60.
7. $15.90.
8. $2.23.

Art. 213

1. Dec. 1, 1906, $309.
2. Feb. 10, 1910, $778.13.
3. Feb. 4, 1911, $172.50.
4. July 2, 1908, $403.33.
5. Dec. 4, 1908, $177.19.

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1. $151.88.
2. $303.

3. $650.
4. $505.10.
5. $378.17.
6. $787.08.
7. $827.47.

Art. 215

2. $483.84.
3. $524.27.
4. $257.31.

Art. 220

1. $19.20.
2. $12.
3. $19.69.
4. $35.
5. $293.75.
6. $134.50.
7. $22.50.
8. $202.50.
9. $35.
10. $3600.
11. $46.25.
12. $60.
13. $3985.

Art. 250

1. $74.62.
2. $147.75.
3. $495.
4. $298.75.
5. $743.12.
6. $1183.50.
7. 1995.
8. $3494.17.
9. $4983.33.
10. $3746.25.

Art. 251

2. Feb. 11, 40 days, $33.50, $5996.50.
3. Oct. 17, 46 days, $4.67, $715.33.
4. Aug. 2, 55 days, $465, $502.85.
5. July 15, 1907, 140 days, $10.97, $459.28.

Art. 252

1. $150.32.
2. $495.08.
3. $739.24.
4. $989.83.
5. $299.46.
6. $7499.25.
7. $593.50.
8. $589.75.
9. $350.02.
10. $456.47.

Art. 255

2. $15, $585.
3. $541.28.
4. $2.55.
5. $2439.02.
6. $446.43.
7. 1¢.
8. $8.88.
9. $200 cash.
10. $17.04.

Art. 267

3. $1831.25.
4. $160.
5. $156.25.
6. $2000.
7. $33531.25.
8. 5%.
9. 4⅛%.
10. $300.

Art. 269

1. $.60.
2. 33⅓%.
3. $5.
4. $750, $825.
5. $2.60, $7.60.
6. 20%.
7. 6¼% loss.
8. 60%.
9. 80%.
10. 44$\frac{4}{11}$%.
11. $1.25.
12. $72.
13. 14%.
14. $75.68, 39.9+%.
15. 31.5+%.
16. 50+%.
17. 58+%.
18. 18⅓%.
19. 4%.
20. $360.

Art. 275

2. $96, $80, $48.
3. $252, $302.40, $201.60.
4. $2000, $600, $1800, $700.
5. $5600.
6. $400.

Art. 284

1. $5263.13.
2. $301.80.
3. $50.04.
4. $801.80.
5. $289.57.
6. $242.63.
7. $118.13.
8. $192.86.
9. $1005.
10. $123.93.
11. $1576.97.
12. $2751.40.
13. $749.25.
14. 650 francs.
15. $51.

Art. 293

1. 17 min. 2 sec.
2. 8 hr. 3 min. 10 sec.
3. 1 hr. 13 min. $3\frac{1}{3}$ sec.
4. 4 hr. 50 min. 3 sec.
5. 8 min. 43 sec.
6. 1 hr. 42 min. 3 sec.
7. 3 min. $1\frac{1}{3}$ sec.
8. 49 min. $42\frac{2}{3}$ sec.
9. 50 min. $21\frac{1}{3}$ sec.
10. 10 hr. 3 min. 2 sec.

Art. 294

1. 63° 48′.
2. 51° 15′.
3. 131° 22′ 30″.
4. 32° 3′ 45″.
5. 93° 48′ 45″.
6. 41° 15′.
7. 18° 52′ 30″.
8. 125°.
9. 86° 26′ 15″.
10. 142° 41′ 15″.

Art. 295

1. 26° 15′ W.
2. 123° W. San Francisco.
3. 38° 15′ W.
4. 11 : 40 A.M.
5. 35 min. fast.
6. 29 min. back.
7. 9 : 47 A.M.
8. 12 : 05 P.M.
9. 8 : 53 A.M.
10. 52° 45′ W.

Art. 300

1. 9 : 00 A.M.
2. 3 P.M.
3. 6 : 00 P.M.
4. 2 : 00 P.M.
5. Noon.
6. 1 : 00 P.M.
7. 3 : 20 P.M.
8. 30 min.
9. 45 min.
10. 24 hr. 35 min.
11. 24 hr. 25 min.
12. 7 da. 17 hr. 30 min.
13. 8 da. 15 hr. 15 min.
15. 15 min. $44\frac{4}{9}$ sec.
16. 8 min. 12.4 sec. faster.
17. 12 hr. 59 min. 54 sec., 13 hr.
18. 2 : 00 A.M. Wed.

Art. 307

1. 8.435 km.; 843500 cm.; 84350 dm.; 843.5 dkm.
2. 64.132 Km.
3. 285 cm.
4. 20 m.
5. 32 m.
6. $14.88.
7. 20.8 m.
8. 46.5 mi.
9. 38.8 mi.
10. (1) 4+ Km. (2) 40+ cm. (3) 3.66 m. (4) 80.5 Km. (5) 1.27 cm. (6) 45.7 m. (7) 274+ m. (8) 805 Km. (9) 20+ cm. (10) 30+ cm. (11) 1.83 m. (12) 30.48 m.

Art. 311

1. $50.23.
2. 112 sq. cm.
3. 159600 sq. m.
4. $1690.
5. 204,633 sq. mi.
6. 3.5 hektares.
7. 3.6 hektares.
8. 70.
9. 20.
10. $30,000.
11. $7.20.
12. 27.87.
13. 358.8.
14. 20.23.
15. 247+.

Art. 315

1. 4000.
2. .15.
3. $39.38.
4. 5.75.
5. 12.
6. $4.
7. 125.5.
8. 30.6.

Art. 319

1. 90.
2. 225.
3. $5.
4. $4.90 gain.
5. $47.94.
6. 120.
7. $9.43.
8. 100.
9. $135.
10. $23.73.

Art. 323

1. .035 oz.
2. 1.102 T.
3. 55.115 lb.
4. 52 kg.

5. 2204.6 lb. 6. $3\frac{1}{4}+$ ct.
7. 2800 kg.

Art. 324

1. 115.
2. 12.45 kilos.
3. 208 mi.
4. 660 km.
5. 89 kg.
6. 1313.
7. 1 hektogram.
8. 20 by 25 cm.
9. 25.6.
10. 6000.
11. 1 m.
12. 500.
13. 1003.
14. Loss $1.64.
15. $1162.50.
16. $1.13.
17. 18.75.
18. 1.8.
19. 137 kg.+.
20. 50.
21. $17+.
22. 19.6 sq.m.
23. 1.3+.

Art. 328

1. 67¢.
2. 25¢; 37¢.
3. $13; $8.63; $7.18.
4. 60, 69, 11, 14, 17, 117.
5. $\frac{1}{6}$.
6. $\frac{3}{8}$.
7. $\frac{14}{15}$, $7.
8. 25¢.
9. $18.35.
10. $5.42; $4.26.
11. $13.80; $9.65; $12.17; $8.46; $8.44; $6.25.

Art. 330

1. $13.53; $9.14.
2. 55¢; 11¢.
3. 5 fr.; 97¢.
4. $5.55; $3.85; $2.70; $2\frac{3}{5}$¢.
5. 3.75 l. or 73¢.
6. 1.25 l. or 24¢.
7. 23.65 lire.
8. 128.25 fr., $24.75.
9. 7.60 fr., $1.47.
10. $23.16.

Art. 333

1. 32¢; 3¢.
2. $2.27; $2.86; 117+ miles.
3. 50 c.
4. 1.50 crowns.
5. 60¢.
6. 19.25 fl., $7.80.
7. $60.75.
8. 84¢.
9. $1.40.
10. $80.

Art. 337

1. 21°+.
2. 37°; 36°+; 41°+.
3. 61° to 68°.
4. 12.8°.
5. 15.5°.
6. 77°.
7. 12°.
8. 23°.
9. −22.8°.
10. 59°; 86°; 113°; 14°; 53.6°; −4°.

Art. 347

1. 3.
2. 12.
3. 5.
4. 30.
5. 6.
6. 2.
7. 38.
8. 120.
9. 21.
10. 56.
11. $4\frac{4}{5}$.
12. 12.
13. 106. 13. 36.
14. 40.
15. 60.
16. 7.
17. 5.

Art. 348

2. 10.
3. 60 length; 40 width.
4. $3 and $2.
5. $720.
6. $1800.
7. 24 yr., 40 yr.
8. $78.82.
9. $2500.
10. $100.
11. $40.
12. 5 ft. 8 in.
13. $1\frac{1}{2}$ days.
14. $1\frac{13}{17}$.
15. $1125 and $875.
16. 5 miles.
17. 50 milk, $12\frac{1}{2}$ water.
18. $320.
19. $3.
20. 6 and 9.
21. 20 and 15.
22. $7020.

Art. 353

1. $x = 4$; $y = 5$.
2. $x = 12$; $y = 5$.

3. $x = 10$; $y = 3$.
4. $x = 2$; $y = 1$.
5. $x = 2$; $y = 1$.
6. $x = 4$; $y = 3$.
7. $x = 3$; $y = \frac{1}{2}$.
8. $x = 9$; $y = 2$.
9. $x = 4$; $y = 10$.
10. $x = 6$; $y = 4$.
11. $x = 4$; $y = 2$.
12. $x = 8$; $y = 16$.
13. $x = 4$; $y = 5$.
14. $x = 11$; $y = 12$.
15. $x = 6$; $y = 4$.
16. $x = 6$; $y = 24$.

Art. 354

1. $x = 16$; $y = 12$.
2. $x = 7$; $y = 3$.
3. $x = 6$; $y = 4$.
4. $x = 72$; $y = 60$.
5. $x = 3$; $y = 2$.
6. $x = 8$; $y = 10$.
7. $x = 15$; $y = 14$.
8. $x = \frac{8}{9}\frac{8}{3}$; $y = 6\frac{1}{1}\frac{2}{3}$.

Art. 355

1. $x = \$6$; $y = \$2$.
2. $\frac{5}{11}$.
3. 13 and 20.
4. 54 and 21.
5. Wheat \$1, rye, \$.80.
6. 23 and 17.
7. 840 men, 160 women.
8. 233 and 142.
9. 81*s.* for sherry; 72*s.* for brandy.
10. *A* \$70; *B* \$30.

Art. 360

2. 4.
3. $\frac{1}{2}$.
4. $\frac{1}{12}$.
5. 12.
6. 9.
7. .9144.
8. 1.09+.
9. .3183; 3.1416.
10. .62+; 1.61.
11. 2.27.
12. 1.101.
13. $\frac{3}{5}$.
14. .1875.
15. $1\frac{1}{2}$.
16. $\frac{1}{4}$ and $\frac{1}{3}$.
17. $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{19}$.
18. 8; .296.
19. 4.
20. 4.
21. $\frac{1}{4}$; 4.
22. $1\frac{1}{2}$; .8.
23. $\frac{3}{5}$; $2\frac{1}{2}$.
24. .06.
25. .80; $1\frac{1}{2}$.

Art. 361

1. 9.
2. 1.5¢.
3. $\frac{1}{2}$ min.
4. \$.375.
5. \$2.18.
6. \$1.425.
7. 1333.33 lb.
8. \$1.125.
9. 15¢.
10. 10.7¢.
11. 63.4¢.
12. \$370.58.
13. .66+¢.
14. \$5.
15. \$20.
16. \$1200.
17. \$2700.
18. 16.
19. \$75.
20. \$120.

Art. 364

1. 75¢.
2. 13.
3. 4 days.
4. \$1680.
5. 162.4 miles.
6. 216 miles.
7. \$345.
8. \$480.
9. 6 oz.
10. 11 bu. 1 pk.
11. \$7320.
12. \$1.64.
13. *A* \$60; *B* \$90.
14. \$10.50.
15. 36 sec.
16. 800.
17. 216.

Art. 366

1. 80 ft.
2. 40 ft.
3. 45 ft.

Art. 367

1. Height 25 ft.; width 40 ft.
2. 150 ft.
3. 95 ft.
4. 50 ft.

Art. 370

2. 1800 lb.
3. 1400 and 1500 lb.
4. 11 lb. 5. 6 ft. from one end.

Art. 371

1. 4000 lb. 2. 1600 lb.

Art. 373

1. 250 lb.

Art. 374

1. 52 lb. 2. 37½ lb.

Art. 380

2. 83 yd. 5. 5 ft.
3. 316 yd. 6. $40.
4. 4 ft.

Art. 381

1. 156.
2. 437.
3. 211.
4. 862.
5. 176.
6. 634.
7. 305.
8. 125.
9. 7.23.
10. 29.541+.
11. 9.32+.
12. 8.45+.
13. 2.085.
14. 6.57+.

Art. 383

1. $\frac{16}{17}$.
2. 2.828+.
3. .4472.
4. 2.45.
5. 1.58+.
6. .88+.
7. 1.89+.
8. .9487.
9. 2.06.
10. 2.64+.
11. .93.
12. .09.
13. 2.6 nearly.
14. 2.91.
15. 111.1+.
16. .61+.
17. $\frac{1}{3}$.
18. .6825.
19. 9.002+.
20. 6.02+.

Art. 386

1. 127.28 ft.
2. 95 in.
3. 39.54+.
4. 63.2+.
5. 366.7+.
6. 21 ft.
7. 25.45 ft.
8. 40 ft.
9. 5 ft.
10. 10 ft.
11. 18.02 ft.
12. 146.8+ mi.
13. *A* 15.98 ft.
    B 10.58 ft.

Art. 396

1. 20 sq. ft.
2. 30.40 sq. ft.
3. 120 sq. in.
4. 94.247 sq. in.
5. 588 sq. in.
6. $39.58.

Art. 399

1. 96 sq. ft.
2. 75.4 sq. ft.
3. 160 sq. ft.
4. 125.66 sq. ft.
5. 179.36 sq. ft.
6. 126.67 sq. ft.
7. 204.2 sq. ft.

Art. 401

1. 314.16 sq. in.
2. 28.274 sq. in.
3. 56.55 sq. in.
4. 235.62 sq. in.
5. $1.31.

Art. 405

1. 240 cu. in.
2. 1178.1 cu. in.
3. 80 cu. in.
4. 392.7 cu. in.
5. 1000 cu. in.
6. 523.6 cu. in.
7. 4.19.
8. 24.48.
9. 69.8 lb.
10. 240.59 lb.

Art. 406

1. 328.
2. 50.
3. 2, 2, 2, 2, 3.
4. 1440.
5. LXIV, CCCXXXV, MDCCCCX.
6. 4250.
7. .00021428+.

8. 7.5.
9. .04.
10. 35.7125.
11. .625.
12. $\frac{7}{16}$.
13. 2.
14. $27\frac{8}{9}$.
15. 844050.
16. $13845\frac{5}{12}$.
17. $33.33.
18. 15%.
19. 50.
20. $2\frac{1}{2}$.
21. $10080.
22. $44800.
23. .03125.
24. 7.5.
25. $2000.
26. $504325.65.
27. $11307\frac{1}{12}$.
28. $4940.
29. 2123.66.
30. $5\frac{25}{144}$.
31. $337\frac{1}{2}$.
32. $1\frac{5}{7}$.
33. 2.
34. $\frac{14}{25}$.
35. $\frac{2}{3}$.
36. $\frac{23}{150}$.
37. $3\frac{3}{4}$.
38. $1.75.
39. .03125.
40. $\frac{15}{16}$.
41. $6708.75.
42. $22.05.
43. $60.
44. 1978 in.
45. $1045.50.
46. 960.
47. 4000 sq. yd.; $195.
48. 12965 gal.
49. $\frac{3}{7}$ qt.
50. $\frac{1}{6}$ bu.
51. .004.
52. $3.53.
53. 85 yr. 11 mo. 22 da.
54. $26\frac{7}{16}$.
55. 337 yd. 1 ft. 7.25 in.
56. 5 gal. 3 qt. 1 pt.
57. 2; 12; 3.
58. 7 A.M.
59. 23° 45′ W.
60. 5 hr. 59 min. 51 sec.
61. 61 ft.
62. $103\frac{1}{3}$.
63. $13324.22.
64. 264 rd. × 264 rd.; $264.
65. 178 bu.
66. $8\frac{1}{2}$ ft.
67. $112.
68. $9\frac{1}{3}$.
69. $15.
70. $8.10.
71. $96\frac{1}{2}$.
72. $697\frac{1}{2}$.
73. 897.7.
74. 881.28.
75. $145\frac{5}{8}$.
76. 4500 bu.
77. $16.
78. Loss $208.33.
79. $66\frac{2}{3}\%$.
80. $16\frac{2}{3}$.
81. $105.
82. $18.98.
83. $27.08.
84. $2.50.
85. P. W. $147.78; T. dis. $2.22.
86. B. dis. $2.25; proceeds, $147.75.
87. $24975.
88. $37.50.

89. $69.93.
90. $243.
91. 31⅛%.
92. $45.
93. $30.
94. $502.50.
95. 4⅘.
96. $503.
97. $485.25.
98. $96.53.
99. 48° 7′ 30″.
100. 49 min. 3 sec.
101. 7524.
102. 466+ miles.
103. 1.52+ m.; 3.22 Km.; 10.16 cm.; 18.29 m.
104. 3.1 mi.; 32.8 yd.; 9.45 in.
105. $4047.
106. 123.55.
107. 4.9.
108. 54.936.
109. 120.
110. 141.88.
111. 283.89.
112. 330.30.
113. $226.80.
114. 110.23 lb.
115. 11662 lb.
116. 4.9 sq. m.
117. 129.50 fr.
118. £ 5 2*s.* 9*d.*
119. 105 marks.
120. 61.70 florins.
121. 21°+.
122. 86°.
123. 14°.
124. Gain $1.20 per yd.
125. 6.
126. 3600 sq. rd.
127. $200; $300.
128. 12¢; 18¢.
129. 27¢; 18¢.
130. $x = 2$; $y = 1$.
131. $x = 4$; $y = 187.5$.
132. 10; 15.
133. 2.
134. 12.
135. 45.
136. 12.
137. $421.88.
138. 115 ft.
139. 1650.
140. 48.
141. 3.
142. 484.
143. .81.
144. .089.
145. 11.66 in.
146. 9 in.

Art. 407

1. 600; 1000.
2. 5 : 8 : 34⅔.
3. 12½.
4. 17.
5. 108.
6. 30.
7. 16.
8. $2156.25.
9. $2487.56.
10. 50%.
11. 20.518 ft.
12. 19.635.
13. $355.33
14. $956.25.
15. .0125.
16. 4 yr. 9 mo. 22 da.
17. $157.81.
18. 864.
19. 586⅔.
20. 118.8.
21. 5½ mi.
22. 3.185 in.
23. 1 A.M. Jan. 3.
24. 2.09.
25. 33⅓%.
26. 33⅓%.
27. $588.24.
28. 16.
29. 50%; 33⅓%.
30. 43¢.
31. 40.
32. 11.31 in.
33. $292.92.
34. $214.29.
35. 21.99.
36. 1000%.

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